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DESIGN OF OPTIMUM TWO-MIRROR ANTENNAS WITH THE OSCILLATION OF R--ETC(U)

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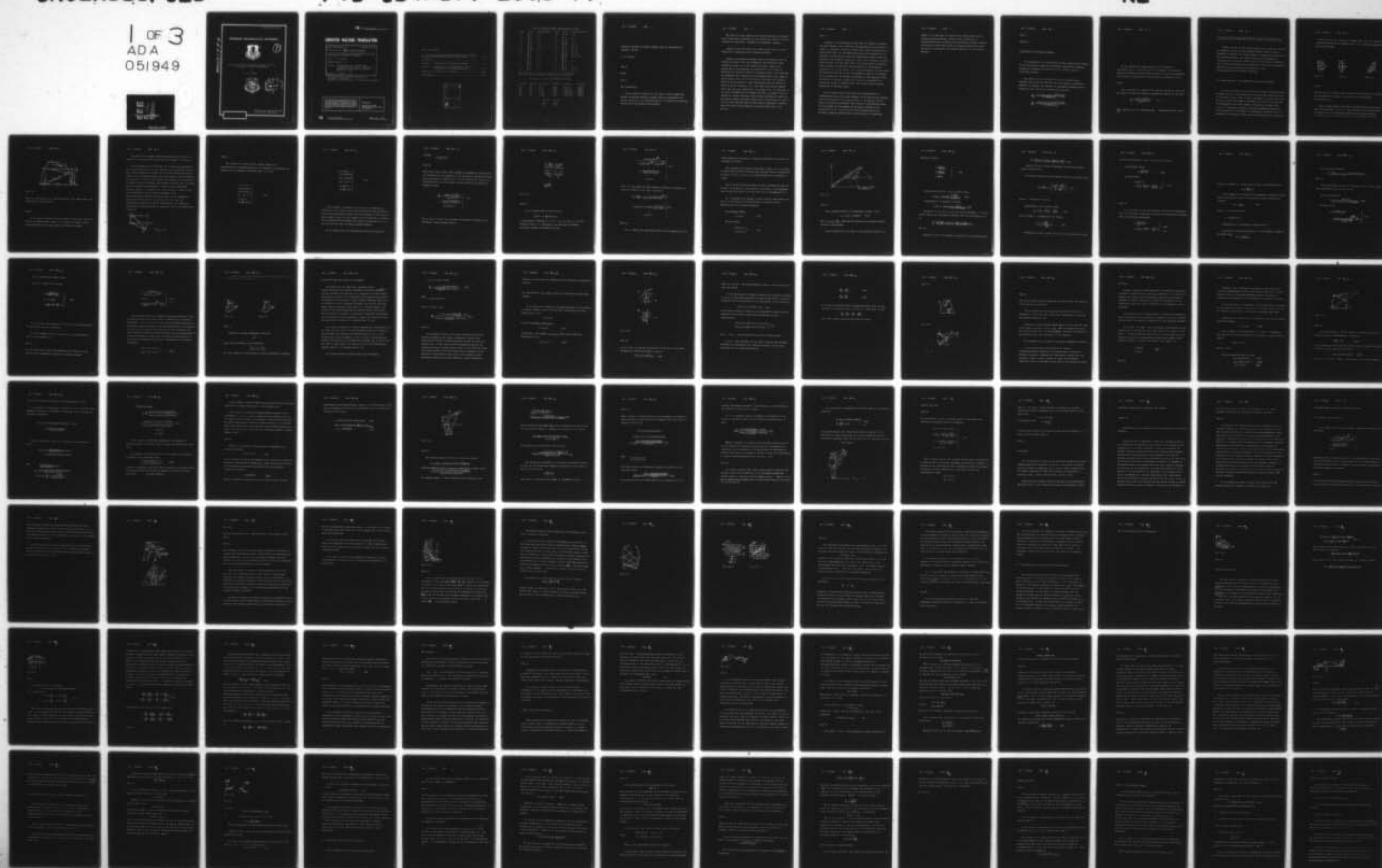

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AIR FORCE SYSTEMS COMMAND

DESIGN OF OPTIMUM TWO-MIRROR ANTENNAS WITH THE OSCILLATION OF RADIATION PATTERN

by

G. K. Galimov

FOREIGN TECHNOLOGY DIVISION

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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

DESIGN OF OPTIMUM TWO-MIRROR ANTENNAS WITH THE OSCILLATION OF RADIATION PATTERN.

G. K. Galimov.

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The introduction

To the specific character of the work of radio engineering devices on movable complexes imposes definite requirements for antenna devices for the target/purpose of the provision for minimum overall sizes and maximum effectiveness.

Is known the large number of the antenna systems of different form. Sufficiently promising for the construction multiple function antennas are two-mirror - aplanatic and parabolic antennas.

Aplanatic antennas possess the substantially higher scanning properties in comparison with parabolic antennas.

However, of aplanatic antennas, just as in parabolic and all monofocal antennas, have the fundamental deficiency/lack which contradicts their use as the scanning antennas. Namely, the aberrations of such antennas are proportional to the angle of deflection of ray/beam. Between in searching sector, all directions are equivalent and a decrease in the amplification at the edges of sector leads to the loss of information. Therefore most adequate would be antenna system at whose aberrations would be uniform on entire sector of scanning, i.e., they would have certain limited value (not more that permissible, for example $\lambda/4$), not depending on the angle of deflection of ray/beam. The same is related to the IR systems of discrete scanning, when antenna form/shapes a series of the widely diverse ray/beams, symmetrically arrange/located relative to axis. Most suitable would be here bifocal anastigmatic antenna, since the known bifocal antennas form pencil beam only in one section.

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Finally, it was repeatedly noted, that the aplanatic antennas, are more accurate, sine condition, were developed for the paraxial region of infinitely fine/thin systems with a small carrying out of source from focus. In wide-angle antennas and the objectives it is necessary to deal with far from such simplified circuits. Thus, for instance, some aplanatic antennas in their output approach a natural limit, having 1:0.6 with beam width 1.1° and the sector of scanning ±10°. Moreover, a significant deficiency/lack in the aplanatic antennas it is, in the general case, large astigmatism, for the value of which sine condition is set no limitations, since it is obtained for a meridian cut. but between astigmatism in large measure limits possibility of scanning antennas and often it is impossible to realize the sector of scanning which is designed only on maximum distortions in meridian plane.

In the present work is not placed the target/purpose of investigating singirg the known versions of the optical-type antennas with the oscillation of radiation pattern - this material can be found in periodical literature. More current is represented another to the problem: the development of the general methods of the construction of the antennas which could optimally solve the problems, appearing before radar in various areas of technology.

namely, it is desirable to construct the unified theory of the scanning optimum antennas, involving both formulation of common/general/total problems and the procedure of their solution. Are presented below some results of research carried out in the direction of optimization and optimum synthesis of the scanning antennas.

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Chapter I.

CALCULATION OF TWO-MIRROR ANTENNAS.

The optimization of multiple-wires antenna requires the creation of the sufficiently powerful and universal mathematical apparatus, suitable for analysis and synthesis of all manifold types of two-mirror antennas.

The creation of the standardized programs requires such equations which could be used also with the numerical calculation methods. For example, the surfaces of axisymmetric antennas can be recorded in the form of two differential equations:

$$\frac{dx_1}{dy_1} = \frac{(x_2 - x_1) + (y_1 - y_2)\,\mathrm{tg}\,\varphi \pm \sec\varphi\sqrt{(x_2 - x_1)^2 + (y_1 - y_2)^2}}{(x_2 - x_1)\,\mathrm{tg}\,\varphi - (y_1 - y_2)};$$

$$\frac{dx_2}{dy_2} = \frac{(y_1 - y_2)\,\mathrm{tg}\,\varphi - (x_2 - x_1) \pm \sec\varphi\sqrt{(x_2 - x_1)^2 + (y_1 - y_2)^2}}{(x_2 - x_1)\,\mathrm{tg}\,\varphi + (y_1 - y_2)}.$$

In the general case these equation are not solved in quadratures, but the application/use, fcr example, of a method of numerical integration (Runge-Kutta) requires sc that the derivatives dy/dx would be expressed in an explicit form as function x and y.

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Most convenient for applying the numerical methods is this path: the surface of auxiliary mirror is assigned by equation of the type

$$\frac{d\rho}{d\varphi} = \rho\frac{\ell(\varphi)\sin\varphi + 2\,\mathrm{tg}\frac{\varphi}{2}(d-\rho)}{2(d-\ell(\varphi)\sin^2\frac{\varphi}{2})}, \qquad (I.I)$$

which contains only one derivative $\frac{d\rho}{d\varphi}$, and characteristic curve $\ell(\varphi)$

can be with any accuracy approximated by interpolotion methods; the surface of auxiliary mirror it is assigned in parametric form.

Further, variety of the antenna systems whose equations obtained in different time and by the different authors, does not make it possible to construct the unified theory of analysis and synthesis of all scanning antennas, for example mirrcr type. Therefore the target/purpose of present chapter lies in the fact that, introducing the of single classification of axisymmetric and axially nonsymmetric type optical-type antennas and obtaining fcr them common/general/total calculated equations.

§1. Generalization of the equations of two-mirrcr antennas.

Is known at present a whole series of circuits two-mirror of the antennas, which are characterized by from each other mutual location in the space of light beams at input (ray/beams of source) and at the output of antenna (collimated ray/beams), and also by the structure of intermediate light beam. namely, are known the circuits (Fig. 1.1-1.3) of Cassegrain, Gregory, axially nonsymmetric systems and their varieties. As is evident, in Cassegrain's circuit at Fig. 1.1 ray/beams of intermediate frame not the transverse axis of system, as

it takes place in the circuits of Gregory (Fig. 1.2). The antennas, constructed according to diagram in Fig. 1.3, include the elements of both these systems.

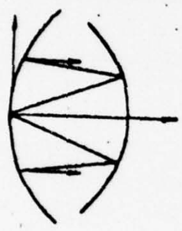

Fig. 1.1.

Fig. 1.2.

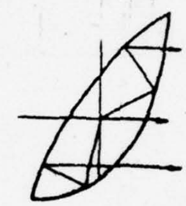

Fig. 1.3.

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In this case, the antennas whose circuits are represented in figures, can be aplanatic, bifocal, they can have special amplitude distribution, etc.

Each of these types of antennas is characterized by from each other also equations, but since these equations fairly complicated, in their form it is sufficiently difficult to establish/install resemblance and the common properties of all two-mirror antennas.

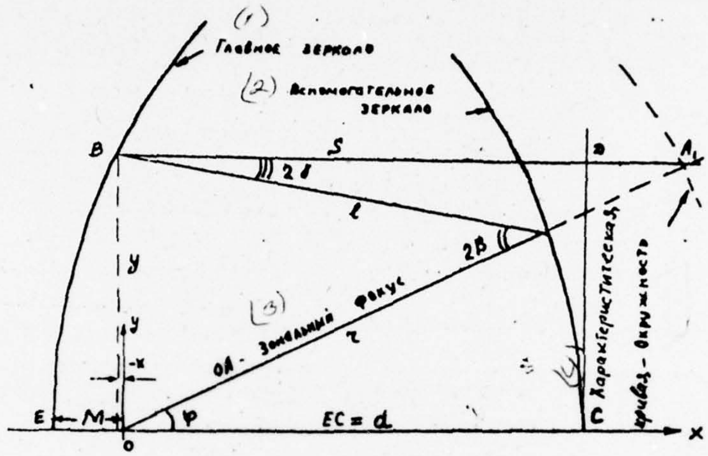

Fig. 1.4.

Key: (1). Main mirror. (2). auxiliary mirror. (3). zonal focus. (4). characteristic curve-circle.

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In the present section we shall attempt to show, how, using the concept of characteristic curve, it is possible to obtain overall relationship/ratios for some types of two-mirror antennas.

Initially let us examine monofocal antennas, and then let us show, as it is possible from them to pass to bifocal to antennas.

Let us examine Fig. 1.4 and Fig. 1.5. On them are represented two versions of course of ray on sections a source-auxiliary mirror - main a mirror-aperture of system. If we do not consider the further course of ray of outside points A and B and angle ϕ, then of these of two versions of course of ray exhaust whole manifold of the patterns of course of ray in different type antennas. Actually, Fig. 1.6 shows whole are possible the versions of course of ray in two-mirror antennas. Hence it is apparent that these versions of course of ray can be subdivided into two classes depending on the relationship/ratio between the current angle of the ray/beams of source and the ordinate (it is more precise, its sign) the corresponding ray/beams at output. namely (Fig. 1.6), positive ϕ corresponds the positive value of ordinate y, and in the second case - negative.

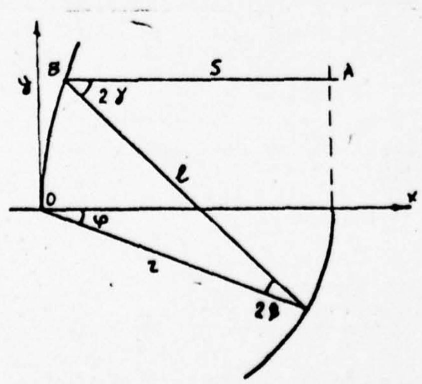

Fig. 1.5.

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Let us paint for these two most typical cases of the chain/network of relationship/ratios, are necessary for obtaining the equations of the surfaces of mirrors (Fig. 1.4, 1.5):

$$\left.\begin{aligned}
&r + l + s = 3d;\\
&r\sin\varphi + l\sin 2\gamma = y;\\
&r\cos\varphi - l\cos 2\gamma = x;\\
&2\gamma = 2\beta - \varphi;\\
&y = f\sin\psi;\\
&\frac{1}{r}\frac{dr}{d\varphi} = \mathrm{tg}\,\beta;
\end{aligned}\right\} \qquad (I.2)$$

$$\left.\begin{aligned}
&r + l + s = 3d;\\
&-r\sin\varphi + l\sin 2\gamma = y;\\
&r\cos\varphi - l\cos 2\gamma = x;\\
&2\gamma = 2\beta + \varphi;\\
&y = f\sin\varphi;\\
&\frac{1}{r}\frac{dr}{d\varphi} = \mathrm{tg}\,\beta.
\end{aligned}\right\} \qquad (1.3)$$

As is evident, in systems (1.2) and (1.3) is observed the difference in the second and the fourth equalities, and precisely, they are distinguished by angle. The third equality in both cases is equal. Based on this, for the purpose of the unification of formulas, let us introduce the following rule: angle ϕ is positive, if it is read off from the axis of antenna counterclockwise.

Let us assume also that characteristic curve is written as

follows:

$$y = |f(\varphi)\sin\varphi|.$$

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Then taking into account these assumption fundamental principles for the derivation of the equations of the surfaces of two-mirror antenna can be presented in system (1.3), and the differential equation of auxiliary mirror and the equation of main mirror let us record in parametric form:

$$\left.\begin{aligned} \frac{dr}{d\varphi} &= r\,\frac{f(\varphi)\sin\varphi + 2\,\mathrm{tg}\,\frac{\varphi}{2}(d-r)}{2\left(d - f\sin^2\frac{\varphi}{2}\right)};\\ x &= \frac{4d(r-d) + f(\varphi)\sin^2\varphi\,(f-2r)}{2[2d - r(1-\cos\varphi)]};\\ y &= |f(\varphi)\sin\varphi|. \end{aligned}\right\} \quad (I.4)$$

Now in order to obtain the equations of two-mirror antenna, it is necessary to only consider angle ϕ.

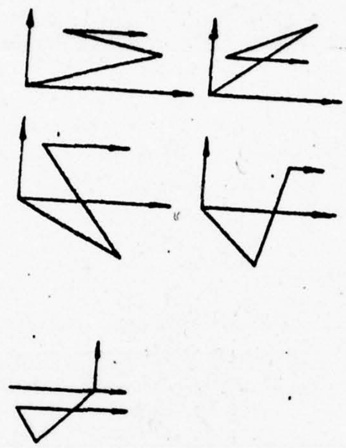

Fig. 1.6.

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So, if characteristic curve has form

$$y = f\sin\varphi, \text{ i.e. } \frac{d}{df}\left[f(\varphi)\right] = 0,$$

a the boundary conditions $y = 0$, $x = 0$, $z = d$ with $\phi = 0$ on the condition that $\phi = 0$, equations (1.4) describe an aplanatic two-mirror antenna of Cassegrain's type:

$$\left.\begin{aligned} r &= \frac{(d \pm M)\left(\cos\frac{\varphi}{2}\right)^{\frac{2d}{f-d}}}{\sin^2\frac{\varphi}{2}\left(\cos\frac{\varphi}{2}\right)^{\frac{2d}{f-d}} + \left(1-\frac{f}{d}\sin^2\frac{\varphi}{2}\right)^{\frac{f}{d-f}}}; \\ x &= \frac{4d(r-d)+f\sin^2\varphi\,(f-2r)}{2\left[2d-r(1-\cos\varphi)\right]}; \\ y &= f\sin\varphi. \end{aligned}\right\} \quad (I.5)$$

If $\phi < 0$, then under the same boundary conditions is obtained an aplanatic antenna of the type of Gregory:

$$\left.\begin{aligned} \frac{1}{r} &= \frac{\sin^2\frac{\varphi}{2}}{d} + \frac{1}{d \pm M}\left[\left(1+\frac{f}{d}\sin^2\frac{\varphi}{2}\right)^{f}\left(\cos^2\frac{\varphi}{2}\right)^{d}\right]^{f/f+d}; \\ x &= r\cos\varphi - \frac{1}{2}\left[2d-r(1-\cos\varphi)\right]\left[1-\left(\frac{\sin\varphi\,(r+f)}{2d-r(1-\cos\varphi)}\right)^2\right]; \\ y &= f\sin\varphi. \end{aligned}\right\} \quad (I.6)$$

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Let us examine one additional method of the calculation of the

airfoil/profiles of mirrors, on defining concretely the location of ray/beams in antenna.

The ray/beam, which emerges from focus F (Fig. 1.7) at an angle ϕ, after being reflected from main and auxiliary mirrors respectively at points A and C, it will go on direct/straight CD in parallel to X-axis.

Let us continue direct/straight CD before intersection with AF at point B. According to the condition of problem, it is considered that is known the value of cut FB, which let us designate by $f(\varphi)$.

Let us designate the lengths of cuts AB and BC respectively by u and ϑ. In this case the airfoil/profiles of mirrors will be determined by the following equations:

the auxiliary mirror

$$\rho = f(\varphi) + u; \tag{I.7}$$

the main mirror

$$\left.\begin{aligned} y &= f(\varphi)\sin\varphi, \\ x &= f(\varphi)\cos\varphi + \vartheta. \end{aligned}\right\} \tag{I.8}$$

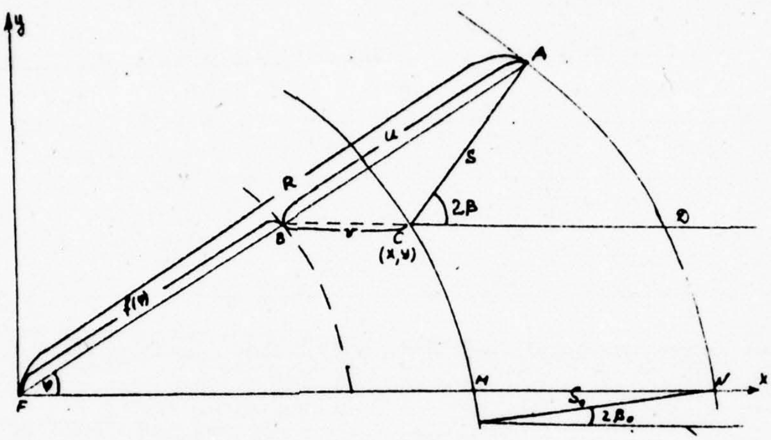

Fig. 1.7.

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Thus, problem consists in determining of value u and ϑ

$$s + u - \vartheta = 2a - (1 - \cos\varphi) f(\varphi). \qquad (I.9)$$

Here $d = s_0 \cos^2 \frac{2\beta_0}{2}$; this cut characterizes the distance between mirrors at data points.

From triangle ABC on the basis of the theorem of sines it is

possible to record:

$$\left.\begin{aligned} & b = \frac{v \sin \varphi}{\sin(2\beta-\varphi)}; \\ & u = \frac{v \sin \alpha}{\sin(2\beta-\varphi)}; \\ & \alpha = 180 - 2\beta. \end{aligned}\right\} \qquad (I.10)$$

Substituting (1.10) in (1.9), we will obtain

$$v = \left[2a - (1-\cos\varphi)f(\varphi)\right] \frac{\sin \frac{\alpha-\varphi}{2}}{2 \sin \frac{\varphi}{2} \cos \beta}. \qquad (I.II)$$

Analogously it is possible to obtain

$$u = \left[2d - (1 - \cos\varphi) f(\varphi)\right] \frac{\sin 2\beta}{\sin\varphi + \sin 2\beta - \sin(2\beta-\varphi)}. \quad (I.I2)$$

Utilizing (1.11), (1.7), (1.8) and using replacement t = tg $\phi/2$, we will obtain the differential equation where by the unknown is tg $\alpha/2$

$$\left[\frac{d}{t} - \frac{tf(t)}{1+t^2}\right] \frac{d\,tg\,\beta}{dt} + tg \frac{\alpha}{2} \frac{d}{dt} \left[\frac{d}{t} + \frac{tf(t)}{1+t^2}\right] + \frac{d}{dt} \left[\frac{f(t)}{1+t^2}\right] = 0. (I.I3)$$

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Equation (1.13) it is possible to rewrite in the following form:

$$\left[\frac{d}{t}-\frac{tf(t)}{1+t^2}\right]\frac{d}{dt}\left[tg\,\beta+\frac{tf(t)}{1+t^2}\right]+\left[tg\,\beta+\frac{tf(t)}{1+t^2}\right]\frac{d}{dt}\left[\frac{a}{t}+\frac{tf(t)}{1+t^2}\right]=0. \quad (I.I4)$$

Equation (1.14) is linear differential first-order equation without absolute term.

The general solution of this equation takes the following form:

$$tg\,\beta+\frac{tf(t)}{1+t^2}=C\exp\left\{-\frac{\frac{d}{dt}\left[\frac{d}{t}+\frac{tf(t)}{1+t^2}\right]}{\frac{d}{t}-\frac{tf(t)}{1+t^2}}\right\}, \quad (I.I5)$$

where C - integration constant.

Simplifying (1.15), we will obtain

$$tg\,\beta=Ct^2\left[\frac{d}{t}-\frac{tf(t)}{1+t^2}\right]e^{J}-\frac{tf(t)}{d(1+t^2)}, \quad (I.I6)$$

where through J is designated the integral

$$J=\int\frac{2t\frac{f(t)}{1+t^2}}{d-\frac{t^2f(t)}{1+t^2}}dt. \quad (I.I7)$$

Substituting (1.15), (1.12), (1.11) in (1.7) and (1.8), we will

obtain the expressions, which lay out of the mirrors:

the auxiliary mirror

$$\rho = \frac{d^2 C(1+t^2)}{dCt^2 + e^{-\vartheta}}; \qquad (I.18)$$

the main mirror

$$\left.\begin{aligned} y &= f(t)\sin\varphi; \\ x &= Ct^2\left[\frac{d}{t} - \frac{tf(t)}{1+t^2}\right]e^{\vartheta} + \frac{t^2 f^2(t)}{d(1+t^2)^2} - d. \end{aligned}\right\} \qquad (I.19)$$

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If is assigned not the characteristic function, but dependence y(ϕ), then the airfoil/profiles of mirrors are conveniently designed, using the following expressions:

$$\left.\begin{aligned} \rho &= \frac{d^2 C(1+t^2)}{dCt^2 + e^{-\vartheta}}; \\ x &= Ct^2\left[\frac{d}{t} - \frac{y(t)}{2}\right]e^{\vartheta} + \frac{y^2(t)}{4a} - d, \end{aligned}\right\} \qquad (I.20)$$

where the integral $\mathfrak{J}$ in this case it takes the following form:

$$\mathfrak{J} = \int \frac{y(t)}{d - \frac{ty(t)}{2}} dt.$$

As an example let us examine the axisymmetric two-mirror system whose characteristic curve is described the curved second order equation

$$f(\varphi) = \frac{f}{1 + \varepsilon \cos \varphi}, \qquad \text{(I.2I)}$$

where f is a focal length;

ε - eccentricity.

Subsequently it is possible to suppose that $f = 1$.

Depending on alternating/variable t, the function $f(t)$ takes the following form:

$$f(t) = \frac{1 + t^2}{(1-\varepsilon)t^2 + (1+\varepsilon)}.$$

Let us compute integral J

$$J = \int \frac{2t\,dt}{t^2\left[\alpha(1-\sigma)-1\right]+d(1+\sigma)} = \frac{1}{d(1-\sigma)-1}\,\ln\left\{t^2\left[d(1-\sigma)-1\right]+d(1+\sigma)\right\}. \quad (I.22)$$

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After substituting (1.22) in (1.18) and (1.19), we will obtain the expressions, which lay out mirrors:

the auxiliary mirror

$$\rho = \frac{Cd^2(1+t^2)}{dCt^2+\left\{t^2\left[d(1-\sigma)-1\right]+d(1+\sigma)\right\}^{\frac{1}{1-d(1-\sigma)}}}, \quad (I.23)$$

the main mirror

$$\left.\begin{aligned} y &= \frac{2t}{(1-\sigma)t^2+(1+\sigma)}, \\ x &= \frac{C\left\{t^2\left[d(1-\sigma)-1\right]+d(1+\sigma)\right\}^{\frac{2d(1-\sigma)-1}{d(1-\sigma)-1}}+\frac{t^2}{d}}{\left[t^2(1-\sigma)+(1+\sigma)\right]^2}-d, \end{aligned}\right\} \quad (I.24)$$

Let us examine some special cases.

If $\delta = 1$, then we will obtain:

$$\left.\begin{aligned} &\rho = \frac{Cd^2(1+t^2)}{t^2(dC-1)+2d};\\ &y = t = \operatorname{tg}\frac{\varphi}{2},\\ &x = \frac{t^2}{4d}(1-dC)+\frac{dC}{2}-d. \end{aligned}\right\} \qquad (I.25)$$

As can be seen from expressions (1.25), by the airfoil/profile of the external mirror is parabola.

If $\delta = 0$, then, as can be seen from (1.21), characteristic curve does not depend on ϕ.

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In this case we will obtain the equations, which lay out of the mirrors of the axisymmetric aplanatic two-mirror systems:

$$
\left.
\begin{aligned}
&\rho = \frac{Cd^2(1+t^2)}{dCt^2 + \left[t^2(d-1)+d\right]^{\frac{1}{1-d}}};\\
&y = \sin\varphi;\\
&x = \frac{1}{(1+t^2)^2}\left\{C\left[t^2(d-1)+d\right]^{\frac{2d-1}{d-1}} + \frac{t^2}{d}\right\} - d.
\end{aligned}
\right\}
\quad (I.26)
$$

Let us return now to the problem of boundary conditions. There is large interest in the case of this combination of the boundary conditions: $\phi = 0$; $x \neq 0$; $y \neq 0$. This case corresponds completely to the special group of antennas - axially nonsymmetric to the antennas in which on the strength of satisfaction of these conditions the axial sections of main and auxiliary mirrors are not perpendicular to the axis of antenna. In this case, can occur two modifications of such antennas depending on the actual combination of the boundary conditions:

$$
\left.
\begin{aligned}
&\varphi = 0;\ x = 0;\ y = Y;\\
&\varphi = 0;\ x = X;\ y = Y.
\end{aligned}
\right\}
\quad (I.27)
$$

In the first case the antenna takes the form as in Fig. 1.8, in the second - as in Fig. 1.9.

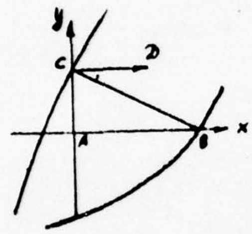

Fig. 1.8.

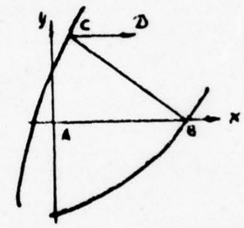

Fig. 1.9.

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Equivalent boundary conditions of the type

$$= Y;$$
$$y = Y$$

they can be, obviously, also conditions:

$$\varphi = \Phi;\ x = 0;\ y = 0;$$
$$\varphi = \Phi;\ x = X;\ y = 0,$$

i.e. the condition of the passage of certain determined collimated

ray/beam through the origin of coordinates.

One should note one additional important special feature/peculiarity of axially nonsymmetric antennas. Function $\Phi(\psi)\sin\psi$ must be connected with angle ϕ_0; it is possible to utilize axially nonsymmetric antennas of the different layout when the direction of the maximum of the diagram of source composes arbitrary angle with the direction of the collimated ray/beams. But if we in this case always take for reference point angle $\phi_0 = 0$ without taking into account of concrete/specific/actual antenna circuit, then can be obtained essential asymmetry in amplitude distribution in aperture. So, in aplanatic axially nonsymmetric antenna the direction of the reference point of angles ϕ is assigned angle $90 + \phi_0$.

The specific character of axially nonsymmetric antennas they are also that which in one and the same antenna occurs of two types of the ray/beams: with positive and negative angle ϕ, i.e., in one antenna there are cell/elements of the circuits of Cassegrain and Gregory, what cannot be in axisymmetric antennas. This is led to the fact that at some angles ϕ is necessary to pass from differential equation from $\phi > 0$ to equation from $\phi < 0$.

As the final result we obtain after some conversions:

for $\phi > 0$ (Fig. 1.10a)

$$\frac{dr}{d\varphi} = r\,\frac{2r - A + 2f\sin\varphi\left[1 - \sin(\varphi_0 - \varphi)\right]}{\dfrac{A\left[1 - \sin(\varphi_0 - \varphi)\right]}{\cos(\varphi_0 - \varphi)} + 2f\sin\varphi\cos(\varphi_0 - \varphi)}, \qquad (I.27a)$$

here

$$y = f\sin\varphi\cos(\varphi + \varphi_0);$$

for $\phi < 0$ (Fig. 1.10b)

$$\frac{dr}{d\varphi} = r\,\frac{y + A - r\left[1 + \sin(\varphi + \varphi_0)\right]\dfrac{\cos(\varphi - \varphi_0)}{1 - \sin(\varphi_0 + \varphi)} - r\cos(\varphi + \varphi_0)}{\left[r\cos(\varphi + \varphi_0) - y\right]\dfrac{\cos(\varphi + \varphi_0)}{1 - \sin(\varphi_0 + \varphi)} + A - r\left[1 + \sin(\varphi + \varphi_0)\right]}. \qquad (I.28)$$

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The important variety of two-mirror antennas they are the bifocal axisymmetric and axially nonsymmetric antenna which form/shape two diverse in space radiation patterns and have two separate foci. In this case, axially nonsymmetric antenna can form/shape radiation pattern (plane front) in any direction, which forms certain angle α with the optical axis of antenna. On the strength of this property and in view of the absence of this substantial limitation as axial symmetry, axially nonsymmetric

antenna it can satisfy the condition of the formation of ray/beams of two foci.

§2. Calculation of the common circuit of a two-mirror optical-type antenna.

As is known, the fundamental designation/purpose of the majority of optical systems lies in the fact that, converting of one wave surface (Fig. 1.11)

$$z_1 = f_1(x_1, y_1)$$

with the amplitude distribution

$$A_1 = F(x_1, y_1) \qquad (I.29)$$

into another wave surface $z_2 = f_2(x_2, y_2)$ with another amplitude distribution on it

$$A_2 = F_2(x_2, y_2). \qquad (I.30)$$

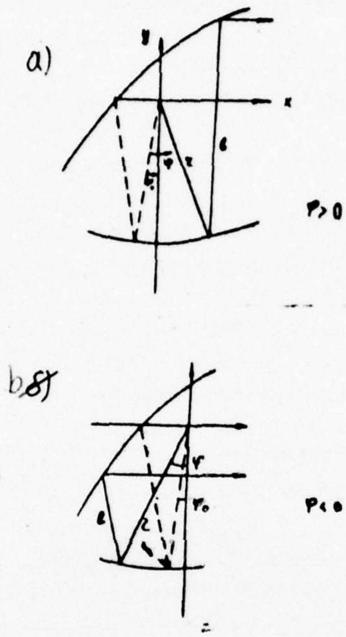

Fig. 1.10.

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In this case, is retained the equality of energies in the beams, determined by these wave fronts z_1 and z_2

$$A_1^2 H_2^{(1)} dx_1 dy_1 = A_2^2 H_2^{(2)} dx_2 dy_2, \qquad (I.31)$$

where $H_2{}^1$ and $H_2{}^2$ - the second quadratic forms of wave fronts of the first and second.

In the plan/layout for common calculation of two-mirror antennas to us it is represented advisable to examine some specific questions, connected with the possibility of the realization of the transition

$$[\varphi_1(x_1, y_1); F_1(x_1, y_1)] \to [\varphi_2(x_2, y_2)]. \qquad (I.32)$$

As is known, two-mirror antenna can be described by system from two equations in the partial first-order derivatives in total differentials:

$$\left.\begin{aligned} dX_I &= \alpha(x_1, y_1, X_I, X_{II})\,dx_1 + \beta(x_1, y_1, X_I, X_{II})\,dy_1; \\ dX_{II} &= \rho(x_1, y_1, X_I, X_{II})\,dx_1 + \vartheta(x_1, y_1, X_I, X_{II})\,dy_1. \end{aligned}\right\} \qquad (I.33)$$

Here X_I and X_{II} are coordinates of the 1st and 2nd mirrors:

x and y - the coordinate of the front of source. The necessary condition for existence of the solution of system (1.33) is the fulfillment of the relationship/ratios:

$$\frac{\partial \alpha}{\partial y_1} = \frac{\partial \beta}{\partial x_1}; \tag{I.34}$$

$$\frac{\partial \beta}{\partial y_1} = \frac{\partial \vartheta}{\partial x_1}. \tag{I.34a}$$

It is easy to see that these relationship/ratios cannot be made strictly in the general case. In fact, in (1.34a) enter derived

$$\frac{\partial x_2}{\partial x_1}, \frac{\partial x_2}{\partial y_1}, \frac{\partial y_2}{\partial x_1}, \frac{\partial y_2}{\partial y_1},$$

which they contain arbitrary amplitude functions.

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Fig. 1.11.

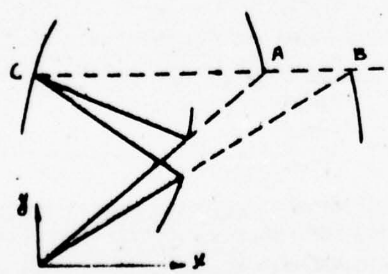

Fig. 1.12

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The form of these functions sometimes can strike such, that equation (1.34) can not be satisfied.

Let us examine the real case, when transition (1.33) it is impossible to carry out, for example when characteristic curve has sectionally-continuous form.

Actually, if characteristic curve consists of two "pieces" (Fig. 1.12), also, of points A and B $y_A = y_B$, then the appropriate point C of the main mirror will hit two ray/beams, which must be reflected in one direction CAB, that, obviously, is not possible.

The analogous case is feasible in axially nonsymmetric antennas.

In the preceding/previous paragraph we examined common/general/total approach on the calculation of two-mirror antennas, generally speaking, the sufficiently limited form, the relating to class systems, forming at output the collimated (parallel) frame of ray/beams for the case of the location of source

at focus.

However, during the investigation of the problem of synthesis, we cannot be limited only by this case. Therefore it is expedient to examine also system with the arbitrary structure of the frame of ray/beams at output, taking into account the possibility of the conversion of the wave fronts, presented in the beginning of this paragraph.

The creation of such antennas makes it possible in principle to pose the problem of the calculation of antennas of with the uniform single error in the sector of scanning and other analogous problems.

Let in Fig. 1.13 FABC - certain ray/beam, characterized by the current angle ϕ and passing outof their system at an angle α to its axis; F - focus of antenna, which lies on the axis. Angle α is the function of angle ϕ. By analogy with conventional systems, which connect ordinate y of point B and angle α with angle ϕ i.e.

$$\begin{aligned} \alpha &= \alpha(\varphi); \\ y &= y(\varphi). \end{aligned} \qquad (I.35)$$

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Function $f(\varphi)$ corresponds according to sense previously introduced characteristic curve, while function $\alpha(\phi)$ is the second steering function. Introducing dependence $\alpha(\phi)$, we thereby is expansible the possibility of the synthesis of two-mirror antennas.

At the same time it is necessary to note that Fermat's condition in the present case must be corrected, on the basis of the fact that the antenna in which $\alpha(\phi) = 0$, form not in parallel light beam.

Therefore in the given case to conveniently use the more overall relationship/ratio:

$$r + l + s = 3d. \qquad (I.36)$$

Here d depends on ϕ moreover with $\phi = 0$ $d(\phi) = d$. Dependence $d(\phi)$ lays out of the wave front which is obtained at output from the antenna

$$s\cos\alpha + x = M,$$

where $M = M(\alpha)$.

Auxiliary equations take the form:

$$r\sin\varphi + l\sin(2\gamma - \alpha) = y; \qquad (I.37)$$

$$r\cos\varphi - l\cos(2\gamma - \alpha) = x; \qquad (I.37a)$$

$$2\gamma - \alpha = 2\beta - \varphi. \qquad (I.37б)$$

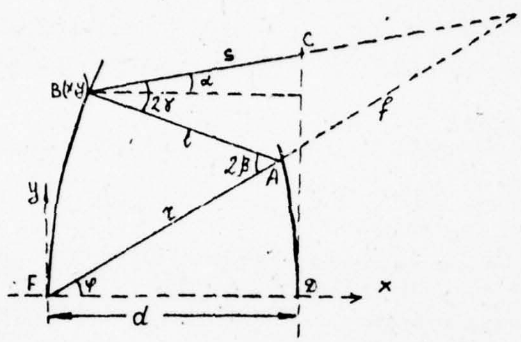

Fig. 1.13.

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The radius-vector r of the surface of auxiliary mirror and angles ϕ and β are determined by relationship:

$$\frac{1}{r}\frac{dr}{d\varphi} = \operatorname{tg}\beta. \qquad \text{(I.38)}$$

It is necessary to note that the function $\alpha(\phi)$ is connected also with crdinate y by the equation of the form

$$f\sin\varphi - y = (f\cos\varphi - x)\operatorname{tg}\alpha, \qquad \text{(I.39)}$$

i.e. in $y = 0$, α-ray $= \alpha(0) = 0$ corresponds to the axis of antenna

and the axial sections of mirrors are perpendicular to axis.

As is evident, in equations (1.38) and (1.39) is contained seven unknowns, according to the number of equations. Let us now make the following conversions:

$$r(\cos\varphi - \cos\alpha) - M + 3d\cos\alpha = l\left[\cos\alpha + \cos(2\gamma - \alpha)\right]; \quad (I.40)$$

$$l = \frac{r(\cos\varphi - \cos\alpha) - M + 3d\cos\alpha}{\cos\alpha + \cos(2\gamma - \alpha)}.$$

Obtained expression (1.40) let us substitute into equation (1.37)

$$r(\sin\varphi - \sin\alpha) + \frac{\sin(2\gamma - \alpha) + \sin\alpha}{\cos\alpha + \cos(2\gamma - \alpha)}\left[r(\cos\varphi - \cos\alpha) - M + \right.$$

$$\left. + 3d\cos\alpha\right] = f(\sin\varphi + \sin\alpha) + 3d\sin\alpha - M\,\mathrm{tg}\,\alpha;$$

here

$$\frac{\sin(2\gamma - \alpha) + \sin\alpha}{\cos\alpha + \cos(2\gamma - \alpha)} = \mathrm{tg}\,\gamma;$$

$$\mathrm{tg}\,\gamma = \frac{f(\sin\varphi + \sin\alpha) + 3d\sin\alpha - M\,\mathrm{tg}\,\alpha - r(\sin\varphi - \sin\alpha)}{r(\cos\varphi - \cos\alpha) - M + 3d\cos\alpha};$$

$$\mathrm{tg}\,\beta = \frac{1}{r}\,\frac{dr}{d\varphi} = \frac{\mathrm{tg}\,\gamma + \mathrm{tg}\,\frac{\varphi - \alpha}{2}}{1 - \mathrm{tg}\,\gamma\,\mathrm{tg}\,\frac{\varphi - \alpha}{2}}.$$

Finally we obtain

$$\frac{1}{r}\frac{dr}{d\varphi}=\frac{f(\sin\varphi+\sin\alpha)+3d\sin\alpha-M\,\mathrm{tg}\,\alpha-r(\sin\varphi-\sin\alpha)+\mathrm{tg}\,\frac{\varphi-\alpha}{2}\left[(r\cos\varphi-\cos\alpha)-M+3d\cos\alpha\right]}{r(\cos\varphi-\cos\alpha)-M+3d\cos\alpha-\mathrm{tg}\,\frac{\varphi-\alpha}{2}\left[f(\sin\varphi+\sin\alpha-r(\sin\varphi-\sin\alpha)\right]-\mathrm{tg}\,\frac{\varphi-\alpha}{2}(3d\sin\alpha-M\,\mathrm{tg}\,\alpha)};\quad (I.4I)$$

$$M=M(\alpha);$$

$$\alpha=\alpha(\varphi).$$

Thus, obtained differential equation for the surface of auxiliary antenna dish, which forms in space the noncollimated light beam.

The equation of the main mirror of this antenna it is possible to express in the parametric form:

$$\left.\begin{aligned} x &= r\cos\varphi-l\cos(2\gamma-\alpha); \\ y &= r\cos\varphi-l\cos(2\gamma-\alpha). \end{aligned}\right\}\quad (I.42)$$

Solution to equations (1.41) and (1.42) requires in the general sense the application/uses of methods of numerical integration on high speed TsVM - [digital computer].

Let us examine, further, common type monofocal antenna, at whose plane front at output forms angle α with vertical axis.

Let in Fig. 1.14 source be arrange/located at point F with coordinates x = 0; y = 0. Let us accept for the reference point of the angles, which determine the ray/beams of source, direct/straight FE, which forms angle ϕ_0 with the negative semi-axis y. Calculated plane front forms angle α with vertical axis and passes through such point C of main mirror, from which the collimated ray/beam is passed through focus F.

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For Fig. 1.14 Fermat's condition must be represented in to following the form:

$$r + l - S = r_0 + l_0 . \qquad (I.43)$$

For the concrete definition of antenna, it is necessary to assign the equation of characteristic curve $f_0 \Theta(\varphi)$, that connects the direction of the beams of source and the position of the corresponding collimated ray/beams y

$$y = f_0 \Theta(\varphi) \sin \varphi . \qquad (I.44)$$

Besides fundamental equations (1.43) and (1.44) we will use also

supplementary relationship/ratios, namely, by the projections of the cuts of ray/beams on the axis of coordinates and by the equation of relation of the angles

$$y = l\cos(2\gamma - 90 - \alpha) - r\cos(\varphi - \varphi_0); \qquad (I.45)$$

$$S\cos\alpha = r\sin(\varphi - \varphi_0) + l\sin(2\beta - \varphi + \varphi) + \frac{S_0}{\cos\alpha} + (y - S\sin\alpha)\operatorname{tg}\alpha; \qquad (I.45a)$$

$$\gamma = \beta + \frac{90 + \alpha - \varphi + \varphi_0}{2}. \qquad (I.45b)$$

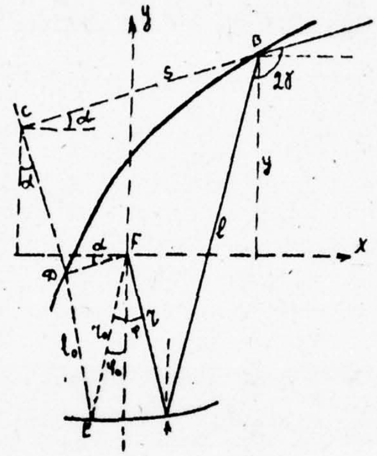

Fig. 1.14.

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We convert equations (1.45) and (1.45a) as follows:

$$(r+l-r_0-l_0)(\cos\alpha+\sin\alpha\, tg\alpha)=r\sin(\varphi-\varphi_0)+l\sin(2\beta-\varphi+\varphi_0)+\frac{S_0}{\cos\alpha}+y\, tg\alpha.$$

From this equation after a series of conversions, we will obtain

$$l=\frac{r\left[\sin(\varphi-\varphi_0)-\cos\alpha-\sin\alpha\cdot tg\alpha\right]+(l_0+r_0)(\cos\alpha+\sin\alpha\cdot tg\alpha)+\frac{S_0}{\cos\alpha}+y\, tg\alpha}{\cos\alpha+\sin\alpha\cdot tg\alpha-\sin(2\gamma-90-\alpha)}.$$

The obtained value l let us substitute into equation (1.45)

$$\frac{\cos(2\gamma - 80 - \alpha)}{\frac{1}{\cos\alpha} \cdot \sin(2\gamma - 90 - \alpha)} =$$

$$= \frac{y + r\cos(\varphi - \varphi_0)}{r\left[\sin(\varphi - \varphi_0) - \cos\alpha - \sin\alpha \cdot tg\,\alpha\right] + (l_0 + r_0)(\cos\alpha + \sin\alpha \cdot tg\,\alpha) + \frac{S_0}{\cos\alpha} + y\,tg\,\alpha}. \quad (I.46)$$

After designating the right side of this equation by M, let us find the expression of angle γ through the parameters of the system

$$\sin^2(2\gamma - 90)\left[(\cos\alpha - M)^2 - (\cos\alpha + \sin\alpha)^2\right] - 2\sin(2\gamma - 90)(\cos\alpha + \sin\alpha)\frac{M}{\cos\alpha} -$$

$$-(\cos\alpha - M)^2 + \frac{M^2}{\cos^2\alpha} = 0.$$

The solution to this equation takes the form

$$\sin(2\gamma - 90) = \frac{2(1 + tg\,\alpha) \pm \sqrt{4(1 + tg\,\alpha)^2 - 4\left[(\cos\alpha - M)^2 - (1 + \sin 2\alpha)\right]\left[\frac{M^2}{\cos^2\alpha} - (\cos\alpha - M)^2\right]}}{2\left[(\cos\alpha - M)^2 - (1 + \sin 2\alpha)\right]}. \quad (I.47)$$

For obtaining the equation of the profile of auxiliary mirror, we will use the differential equation cf standard to the curve by which we used earlier

$$\frac{1}{r}\frac{dr}{d\varphi} = tg\,\beta$$

Here angle β is connected with angle γ by equation (1.47).

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Then, in order to express angle β by the parameters of system, we make a series are converted, after designating the right side of equation (1.47) by B:

$$\sin(2\gamma - 90) = \sin[2\beta + (\varphi_0 - \varphi + \alpha)] = B;$$

$$4E^2(\sin^2\beta - \sin^4\beta) = (B-F)^2 + 2(B-F)F\sin^2\beta + F^2\sin^4\beta;$$

$$\sin\beta = \sqrt{\frac{-[2(B-F)-4E^2] \pm \sqrt{[2(B-F)-4E^2]^2 - 4(4E^2+F^2)(B-E)^2}}{2(4E^2+F^2)}}. \quad (I.48)$$

Here

$$E = \cos(\varphi_0 - \varphi + \alpha);$$
$$F = \sin(\varphi_0 - \varphi + \alpha).$$

We finally obtain the differential equation of profile of the auxiliary mirror

$$\frac{1}{z}\frac{dz}{d\varphi} = \operatorname{tg}\arcsin\sqrt{\frac{-[2(B-F)-4E^2] \pm \sqrt{[2(B-F)-4E^2]^2 - 4(4E^2+F^2)(B-F)^2}}{2(4E^2+F^2)}}. \quad (I.49)$$

This equation with the substitution in it of values E, F, B, M

contains the unknown parameter y and the angle α, which determines the position of plane front in space.

If $\alpha = 0$, equation (1.49) is reduced to the equation of the profile of auxiliary mirror with the arbitrary characteristic of curve

$$\frac{1}{r}\frac{dr}{d\varphi} = \frac{A - r\left[1+\sin(\varphi+\varphi_0)\right]\frac{\cos(\varphi+\varphi_0)}{1-\sin(\varphi_0+\varphi)} - r\cos(\varphi+\varphi_0)+y}{\left[r\cos(\varphi+\varphi_0)-y\right]\frac{\cos(\varphi+\varphi_0)}{1-\sin(\varphi_0+\varphi)} + A - r\left[1+\sin(\varphi+\varphi_0)\right]}. \quad (I.50)$$

Above we examined the systems which contained optical axis and for the location of separate ray/beams were not superimposed any limitations. Let us examine now one particular, but important for practice case when the ray/beams of system at output are subordinated to certain concrete/specific/actual law (Fig. 1.15).

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Let certain ray/beam OBAC emerge certain source, outlying into arbitrary point with coordinates (A, m) and passes through the extreme points of auxiliary and main mirror $B(x_B, y_B)$ and $A=(x_A, y_A)$. Let at these points ray/beam OBAC be characterized also by the angles $\theta_0, \beta_B, \varphi_B, \varphi_A, \gamma_A, \xi_0$.

The coordinates of point $B(\varphi_B, r_B, \beta_B)$ are connected by system of equations:

$$\left.\begin{aligned} r_B \sin\varphi_B &= y_A + (r_B\cos\varphi_B - x_A)\,\mathrm{tg}\,\xi_0; \\ \frac{1}{r_B}\,\frac{dr_B}{d\varphi_B} &= \mathrm{tg}\,\beta_B. \end{aligned}\right\} \qquad (I.5I)$$

This system assigns also tangent inclination at points B and A; therefore the angle which forms the cut of ray/beam AC with the undeflected ray/beam, there can be found from the relationship/ratio

$$\zeta = 2\beta_B - 2\varphi_B + \theta.$$

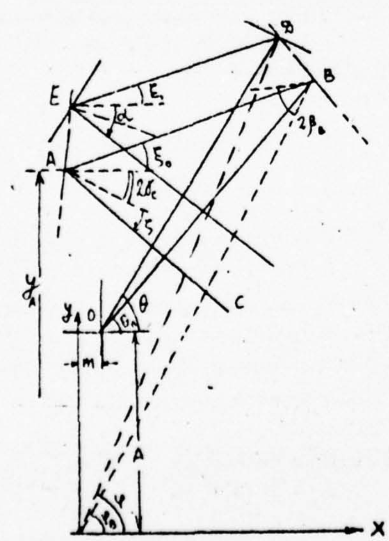

Fig. 1.15.

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The coordinates of point A (extreme point) of main mirror are connected by analogous system of equations:

$$\left.\begin{aligned} & y_A = r_B \sin\varphi_B - (r_B\cos\varphi_B - x_c)\,\mathrm{tg}\,\xi_0, \\ & \frac{1}{r_A}\frac{dr_A}{d\varphi_A} = \frac{f(\varphi_A)\sin\varphi_A + 2\,\mathrm{tg}\frac{\varphi_A}{2}(d - r_A)}{2\left(d - f(\varphi_A)\sin^2\frac{\varphi_A}{2}\right)}; \\ & x_A = \frac{4d(r_A - d) + f(\varphi_A)\sin^2\varphi_A\,(f(\varphi_A) - 2r_A)}{2\left[2d - r_A(1 - \cos\varphi_A)\right]}, \\ & y_A = f(\varphi_A)\sin\varphi_A. \end{aligned}\right\} \quad (I.52)$$

Let us assume further that besides ray/beam DBAC, characterized by angle θ, there are another ray/beams, which emerge at angles $\theta > \theta_n$ moreover on the directions of these ray/beams at output from antenna superimposed certain limitation. In this case, can seem two characteristic cases:

$$2\gamma_A + \zeta = \alpha < A;$$

$$2\gamma_A + \zeta = \alpha > A,$$

where A - the angle, at which emerges the system the ray/beam, falling into certain subsequent point E of the main mirror: in the first case

$$\alpha = (2\gamma_c + \zeta)\left(1 + \frac{y_c}{y_A}\right)M,$$

in the second case

$$\alpha = \frac{M(2\gamma_c - \zeta)}{1 + \frac{y_c}{y_A}}.$$

In both cases function M is selected so that with the growth y_c ($y_c > y_A$) value α would approach value A.

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Conclusion.

For the practical studies of the two-mirror antennas can be used common/general/total equations (1.18) and (1.19), which in special cases are reduced to equations (1.5), (1.6) so forth. Antennas with the complex structure of light beam at output are described by equations (1.41), (1.42), (1.45)-(1.49), (1.51), (1.52).

Apparently, the broadest class of antennas can be described by equations (1.41), (1.42) taking into account the necessary boundary

conditions, which define concretely this antenna.

Chapter II.

Optimization of the parameters and the synthesis of the scanning antennas.

Present chapter is dedicated to posing of the question and to the selection of the methods of the solution of the problem of the optimization of the known parameters of this type antennas. The optimum scanning antenna we will call such antenna which possesses minimum axial size/dimension at given diameter, the sector of scanning, wavelength and during permissible distortion of wave front. In the case of the axially nonsymmetric antenna it is possible to talk about antenna with minimum space with this radiating aperture. In this case, by the parameters, are understood diameter D, focal length f and the apical cut M, expressed in the portions of the axial size/dimension d, and also focal curve. Thus, optimization unlike the procedure of optimum synthesis does not imply a change in characteristic curve, but thereby also form by the antenna of system. Analytically this problem is reduced to finding of the extremum of

the function of quality, but not the extreme function, which minimizes functional in the problem of optimum synthesis.

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In chapter is not carried out the proof of the fact that the scanning properties of antennas depend on their parameters, since this was already shown in a series of the preceding/previous works, for example in [1]. Therefore in essence is made backstop to the analysis of mathematical methods and the selection of the function of quality. It is shown, that the selection of the fundamental method of optimization is dictated by the complexity of the equations of optical-type antennas: in the general case these are the differential equations whose solution is unknown. But if we the solution in the quadratures of this equation obtain is possible, then the equation of the intersection of arbitrary ray/beam and surface proves to be equation with fractional powers (as in the aplanatic antennas). In connection with this as the basis of the gradient method of optimization, just as the method of least squares, must be placed the numerical iterative calculation methods on high speed TsVM [ЦВМ - digital computer].

As the function of quality, which is the criterion of the optimum character of system, is selected the function of

root-mean-square phase error along entire front.

§ 1. Posing of the question.

As is known, the two-mirror aplanatic antennas, described by the equations:

$$r = \frac{(d \pm M)\left(\cos\frac{\varphi}{2}\right)^{\frac{2d}{f-d}}}{\sin^2\frac{\varphi}{2}\left(\cos\frac{\varphi}{2}\right)^{\frac{2d}{f-d}} + \left(1-\frac{f}{\,}\sin^2\frac{\varphi}{\,}\right)^{\frac{f}{d-f}}};$$

$$x = \frac{4d(r-d) + f\sin^2\varphi(f-2r)}{2[2d - r(1-\cos\varphi)]};$$

$$y = f\sin\varphi$$

depending on the relationship/ratio of the parameters f, d and M they can have different shape of surface (Fig. II.1). In these figures is conditionally carried out the general horizontal, which corresponds to certain diameter of antenna.

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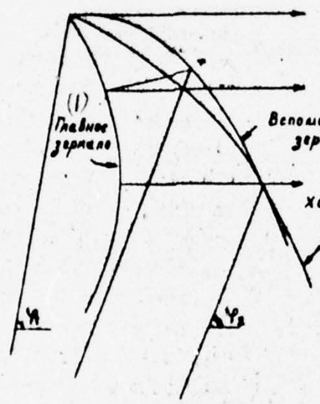
It is evident that the two-mirror aplanatic antenna in the general case can have any axial size/dimension of d, any apical cut M and any

focal length f . In each individual case the antenna will have different curvature, and the different scanning properties, i.e., from relationship/ratios f, d, M depend the distortions which appear in aplanatic antennas with the deflection of the beam or the carrying out of source from focus.

For the illustration of this property Fig. II.2 and II.3 gives picture of course of ray in antennas with different sense $f/D (M=0)$ with the deviations of the falling/incident flat/plane wave front to one and the same angle $\alpha = 40°$.


(1)
Главное зеркало
(2)
Вспомогательное зеркало
(3)
Характеристическая кривая

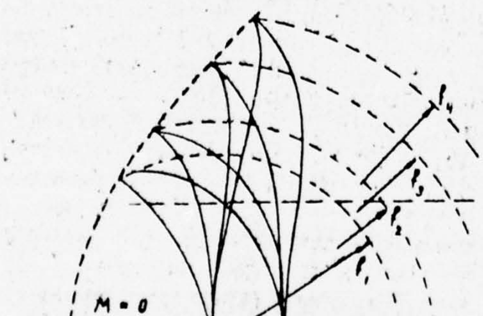

M = 0

Fig. II.1.

Key: (1). Main mirror. (2). auxiliary mirror. (3). characteristic curve.

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As is evident, in Fig. II.2 occur large distortions, since f/d the antenna being investigated differs significantly from optimum value. Here distortions so big that the ray/beams, reflected from main mirror, not at all fall on auxiliary mirror. In Fig. II.3 distortions are less, since f/d antenna is close to optimum.

This diversity of the forms of airfoil/profiles, as in Fig. II.1, not it is characteristic for other forms of optical-type antennas, for example aplanatic lenses, since of lenses $M \neq f$, a thickness it is always desirable to have minimum. This is the otavite before the developer of aplanatic two-mirror antennas the problem of the selection of optimum, i.e., most adequate/approaching for the present instance of version.

Of what does consist the specific character of question? In view of the presence of the large number of independent variables in given antenna to the system, characterized by the diameter of aperture D

and by the permissible phase error Δl_{max} at the edge of the sector of scanning, real phase error can in the general case be more than or less than permissible.

In the first case this means that the antenna has the small axial size/dimension of d or large apical cut (+M), commensurable with d; in the second case of d greatly or apical cut (-M) is great in absolute value.

It is easy to see that the unjustified increase in the axial size/dimension of antenna is led to essential complication of its construction.

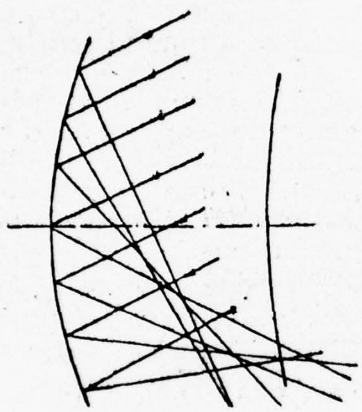

Fig. II.2.

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Let us examine most common/general/total [illeg.] of the dependence of the parameters $\frac{\Delta L_{max}}{d}$, $\frac{D}{d}$, $\frac{f}{d}$, [illeg.] it is possible to present in the form of the curve/graph of Fig. II.4, constructed for the concrete/specific/actual angle of deflection of radiation pattern. In this figure is obtained the dependence of phase error $\frac{\Delta L_{max}}{d}$ on f/d for antennas with different sense f/d It is evident that to each value D/d corresponds certain determined sense f/D, in which $\frac{\Delta L_{max}}{d}$ it has smallest value.

The physical sense of the extreme form of curve/graphs (Fig. II.4) consists of following.

As can be seen from integral curve equations (I.5)-(I.6), to each value f/d corresponds certain maximum/overall diameter $(D/d)_{max}$, in which it occurs the intersection of the airfoil/profiles of main and auxiliary mirrors. In this case, in intersection region, occurs also an increase in the curvature of mirrors. Therefore antennas with the diameter, close to $(D/d)_{max}$, have distortions even somewhat more than antenna with the same diameter, but when larger f/d. Therefore to each D/d corresponds at first decay in value $\frac{\delta L_{max}}{d}$ in function f/d up to certain $(f/d)_{\text{ont}}$, and then an increase in the aberrations proportional to growth f/d.

It should also be noted that the maximum/overall diameter

$$D_{max} = 4f\sqrt{\frac{d}{f}}\sqrt{1-\frac{f}{d}}$$

depends on f/d and therefore on the curve/graphs of Fig. II.4, the initial left points of curves correspond thereby to minimum values f/d in which it can be obtained this relationship/ratio D/d.

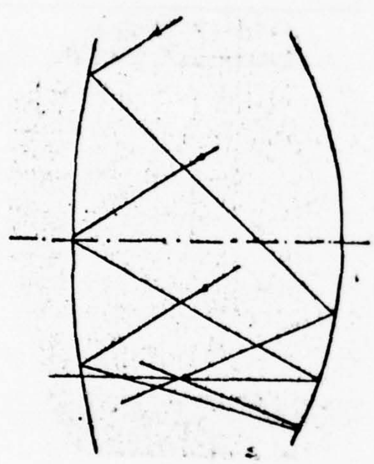

Fig. II.3.

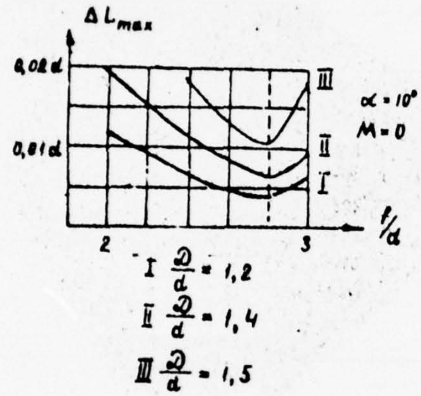

Fig. II.4.

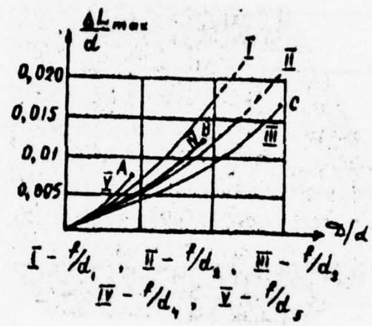

Fig. II.5.

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Are analogously constructed the curve/graphs of Fig. II.5. Here along the axis of abscissas, are deposit/postponed the diameters of antennas, and phase errors are found in the function of different f/d

$$f/d_1 < f/d_2 < f/d_3 < \ldots,$$

moreover as the constant parameter is undertaken diameter D, but not the axial size/dimension d as in Fig. II.4. Points A, B, C on curve/graphs $f/d_5, f/d_4, f/d_3$ correspond D_{max}; the broken sections of curve/graphs $f/d_{1,2}$ they mean that $(D/d)_{max}$ these curves they lie/rest beyond the limits of the values in question.

From Fig. II.5 is also observable the extreme character of the dependence

$$\frac{\Delta L}{D} \rightarrow \frac{f}{d}.$$

Actually, at the selected value ΔL/D and the angle of deflection of radiation pattern α, can be found the maximum value D/2d, to which corresponds certain sense f/d .With others D/d the error ΔL/D can be even the less permissible value, but this so on ratio D/d will also be less than optimum (very "thick" antenna).

One should emphasize that the extreme character of dependence ΔL on the parameters of antenna is typical only for two-mirror antennas with both nonplanar mirrors (parabolic antenna it can be considered as two-mirror antenna with the flat/plane auxiliary mirror). The phase error in single-reflector antenna takes (schematically) this form, as in Fig. II.6, i.e., ΔL is unambiguously connected with the focal distance of antenna about given diameter.

The extreme character of the dependence $\Delta L_{c.кб}$ on the parameters of antenna forces to reexamine approach to the methods of assignment of antennas. Let us examine several examples.

1) It is assigned: the diameter of antenna D, maximum error $\Delta L_{c.кб}$ and the sector of scanning. If antenna must only satisfy these requirements, then, according to Fig. II.5, are sufficient to take $D < D_{max}$, and d such, so that would provide the condition $\Delta L_{c.кб} < \Delta L_{c.кб(доп)}$.

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For the construction of optimum antenna, is necessary unambiguous solution, which will correspond D/d = max. in condition $\Delta L_{c.кб} = \Delta L_{c.кб(доп)}$.

2) It is assigned: the diameter of antenna D, maximum error $\Delta L_{c.\kappa\delta}$ and the requirement so that the sector of scanning would be maximum. This formulation of the problem does nct provide unique solution. Actually, if the axial size/dimension of antenna d is not limited, then can be provided any ratio $\theta/2\alpha$ (beam width to the sector of scanning), which will only ensure the geometry cf antenna - the ray/beams, reflected from main mirror, they must hit to auxiliary mirror.

§ 2. Optimization in initial approach/apprcximation.

Under term "solution of the problem of the optimization of optical system" let us imply the process of finding such system at whose combination of the known parameters d, f, D, M provides the minimum of the standard deviation of phase error $\Delta L_{c.\kappa\delta}$ with respect to entire aperture, or the minimum of maximum deviation from the assigned magnitude. In this case, one should consider that the problem to optimum does not assume finding the parameters of the antennas, which ensure the maximum sectcr undistorted scanning (i.e. the minimum $\Delta L_{c.\kappa\delta}$ without any limitations). In a number of cases, it is to important construct the antenna system which with the assigned sector of scanning and the permissible value of aberrations

must have minimum axial size/dimension.

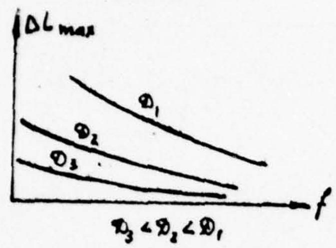

Fig. II.6.

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A) Sine condition [2].

The first task to extremum for scanning systems (in initial approach/approximation) was solved during the derivation of sine condition. In this case by differentiation of Seidel's eikonal $S(y_0, z_0, \eta, \zeta)$ it is possible to determine those changes which undergo the coordinate of the points of intersection of ray/beam with image plane (y_0, z_0, Fig. II.7) and with the plane of the pupil of input η, ζ in comparison with their values, calculated according to the dioptrics of Gauss. So, if we for Schwartzschild's eikonal find derivative in the form

$$dW = y_0\left(d\eta_0 - \frac{M_0}{\eta_0\lambda_0^2}\,dy_0\right) + z_0\left(d\zeta_0 - \frac{M_0}{\eta\lambda_0^2}\,dz_0\right) -$$
$$- y_1\left(d\eta_1 - \frac{M_1}{\eta_1\lambda_1^2}\,dy\right) - z_1\left(d\zeta - \frac{M_1}{\eta_1\lambda_1^2}\,dz_1\right) \qquad (\text{П}.2)$$

and to consider that some members in right side (II.2) are total differentials, for example

$$-\frac{M_0}{\eta_0\lambda_0^2}\left(y_0dy_0 + z_0dz_0\right) = -\frac{M_0}{2\eta_0\lambda_0^2}\,d\left(y_0^2 + z_0^2\right),$$

then instead of W we will obtain function S - Seidel's eikonal

$$S = W + \frac{M_0}{2\eta_0\lambda_0^2}\left(y_0^2 + z_0^2\right) - \frac{M_1}{2\eta_1\lambda_1^2}\left(y_1^2 + z_1^2\right) + y_0(\eta_1 - \eta_0) + z_0(\zeta - \zeta_0).$$

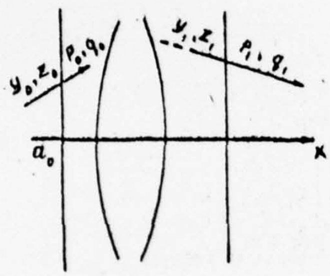

Fig. II.7.

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From equation (II.2) it follows

$$dS = (\eta_1 - \eta_0)\,dy_0 + (\zeta_1 - \zeta_0)\,dz_0 + (y_0 - y_1)\,d\eta_1 + (z_0 - z_1)\,d\zeta_1$$

or in another form

$$\eta_1 - \eta_0 = \frac{dS}{dy_0}; \quad y_1 - y_0 = -\frac{dS}{d\eta_1};$$
$$\zeta_1 - \zeta_0 = \frac{dS}{dz_0}; \quad z_1 - z_0 = -\frac{dS}{d\zeta_1}. \qquad (\text{II}.3)$$

Thus when is assigned the function of Seidel's eikonal $S(y_0, z_0, \eta_1, \zeta_1)$ i.e. it is assigned the path of the ray/beam, passing through the given point of object space and the given point in the plane of the pupil of output, then of it it is possible by differentiation to determine those changes which undergo the coordinate of the points of

intersection of ray/beam with image plane and the plane of the pupil of input in comparison with their values, calculated according to the dioptrics of Gauss. Equations (II.3) give the representation of a change in the coordinates of real focus in comparison with the coordinates of Gaussian focus. In order to obtain now sine condition, let us find the extremum of this function, after taking the second derivative of eikonal in terms of coordinates and after equating to its zero. For this, according to Schwartzschild, let us make the following conversions: of equation (II.3) let us compose four derivatives of the second order of S in terms of each of variables y_0, z_0, η_1, ζ_1. Differentiating with respect to η_1 and y_0, we will obtain:

$$\left.\begin{aligned} &1-\frac{d\eta_0}{d\eta_1}=\frac{d^2S}{dy_0\,d\eta_1}; \quad \frac{dy_1}{dy_0}-1=-\frac{d^2S}{d\eta_1\,dy_0};\\ &-\frac{d\zeta_0}{d\eta_1}=\frac{d^2S}{dz_0\,d\eta_1}; \quad \frac{dz}{dy_0}=-\frac{d^2S}{d\zeta_1\,dy_0};\end{aligned}\right\} \quad (\text{II}.4)$$

differentiation with respect to ζ and z_0 gives:

$$-\frac{d\eta_0}{d\zeta}=\frac{d^2S}{dy_0\,d\zeta}; \quad \frac{dy_1}{dz_0}=-\frac{d^2\zeta}{d\eta_1\,dz_0},$$

$$1-\frac{d\zeta_0}{d\zeta_1}=\frac{d^2S}{dz_0\,d\zeta_1}; \quad \frac{dz_1}{dz_0}-1=-\frac{d^2S}{d\zeta_1\,dz_0}.$$

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Knowing these derivatives, let us determine now condition which must satisfy Seidel's eikonal so that the points on the axis in the plane of object/subject and images would form stigmatic pair (image of point it is obtained sharp). According to this requirement, if $y_0 = z_0 = 0$, then must follow which $y_1 = z_1 = 0$, moreover on this path ray/beam must pass of one point into another, i.e., independent of values η_1 and ζ_1. Then from (II.3) follows that the equation:

$$\left(\frac{ds}{d\eta_1}\right)_{\substack{y_0=0\\z_0=0}}=0;\ \left(\frac{ds}{d\zeta_1}\right)_{\substack{y_0=0\\z_0=0}}=0 \qquad (\text{II}.5)$$

must be satisfied at any values $\eta_1 \zeta_1$ Then let us require so that not only the points on the axis in the plane of object/subject, but also their surrounding cell/elements of surface would be reflect/represented punctually. Analytically this condition means that the condition of the dioptrics of Gauss $y_1 = y_0$, $z_1 = z_0$ (real focus coincides with focus of Gauss) it must be fulfilled not only at points $y_0 = y_1 =$ by 0; $z_0 = z_1 = 0$, but also for the low values y_0, z_0, with an accuracy down to the terms of higher orders, i.e., that the equations

$$\frac{dy_1}{dy_0}=\frac{dz_1}{dz_0}=1;\quad \frac{dy_1}{dz_0}=\frac{dz_1}{dy_0}=0$$

must be satisfied by values $y_0 = z_0 = 0$ at arbitrary values η and ζ_1:

$$\frac{d\eta_0}{d\eta_1}=\frac{d\zeta_0}{d\zeta_1}=1;\quad \frac{d\eta_0}{d\zeta_1}=\frac{d\zeta_0}{d\eta_1}=0.$$

These equations can be integrated, moreover constant integrations will be determined by the facts that the ray/beam, which coincides with axis, does not undergo refraction, i.e., with $\eta_0 = \zeta = 0$ must be $\eta_1 = \zeta_1 = 0$. Thus, we obtain

$$\eta_0 = \eta_1; \ \zeta_0 = \zeta_1. \qquad (\Pi.6)$$

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This means that if two perpendicular to the axis of the cell/element of the surfaces, which surround stigmatic to point on the axis $y_0 = z_0 = 0$ and $y_1 = z_1 = 0$, mutually reflect/represent each other, then coordinates in the equation of Seidel's eikonal η, ζ must be equal for each pair of the conjugated/combined ray/beams, passing through both stigmatic points, i.e., these ray/beams they must encounter the pupils of input and output at the identical given distance from axis.

For the majority of optical tasks, the sine condition is exhausting for the adjustment of coma, since in optics thus far still does not stand so sharply the question of optimization for each concrete/specific/actual case. However, the application/use of ultrapowerful-light wide-angles lens already is placed on agenda and

this question.

By analogy with the given method it seems to us that the task of optimization in parameter it must be sclved alsc in the plan/layout for finding the extremum of function $ds(y_1; x_1; x_2; y_2; A)$.

Here x_1, y_1 are coordinates of the intersection of ray/beams with focal plane; x_2, y_2 - the coordinate cf intersection with plane cf aperture; A - involves the constants of system.

Differential ds includes variations in Seidel's eikonal with respect to the coordinates of system, namely, zcnal aberrations, i.e., the deviation of the real coordinates of ray/beams from their ideal (in the absence of aberrations) cf value.

Search for the second derivative, i.e., finding the extremum of function ds according to parameters, has a series of special feature/peculiarities in comparison with the analogous procedure, used during the derivation of sine condition. Specifically,, abbe sine condition is obtained for the case infinitesimal deviations of representative point from the optical axis cf system; the derivation of this condition is implied no concrete/specific/actual diagram of optical system (lens, mirror, axis either axially nonsymmetric), and that more is not considered such parameters as axial size/dimension

d, diameter D/d, the apical cut M/d or focal length f/d and the angle of the oscillation of radiation pattern θ.

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Furthermore, derivation itself is based on the approximation methods of the theory of the aberrations of the third order: in this approach/approximation of the surface of system can be given not higher than second order equation (sphere, paraboloid, hyperboloid).

Let us examine another series of the analogous ways of optimization in an example of the investigations of Herzberg, such, as analysis of the conditions of the sharp image of the curve of line and element of volume. The derivation of these conditions is based on cosine law.

E) The condition of cosines [3].

Being turned to the derivation of cosine law, let us visualize that in object space, in medium with refractive index n, as is conveniently arrange/located the element of line dl, a in image space, in medium with refractive index n', is found the element of

line $dl' = A'A'_1$, conjugate/combined with cell/element dl. It is necessary to refine: here word goes with coupling in the sense of optics of Gauss; in this case the point image of each point of cell/element dl at the appropriate point of cell/element dl' it can and not be. Only for conjugate points A and A' we will assume the condition of point image carried out, in consequence of which must be correctly the expression (Fig. II.8).

$$AA' = const_1. \qquad (\Pi.7)$$

We assume, therefore, that optical path length is constant along all ray/beams, which connect points A and A'. Our task consist in point image being propagated to all points of element dl, and consequently and to point A_1.

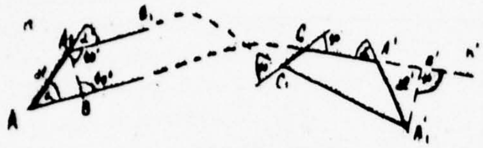

Fig. II.8.

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Let us visualize further one of the ray/beams, which connect according to condition AA' = const, point A and A', for example ray/beam ABCA', which passes in its path through optical system (not shown on drawing) and the forming with cell/element dl angle α, but with cell/element dl' - angle α'. Let us conduct through point A_1 ray/beam $A_1B_1C_1A'_1$, parallel to ray/beam AB in space of objects. In image space, ray/beams CA' and C_1A' not to, but they cross themselves, without intersecting.

In object space, let us drop/omit from point A_1 perpendikul4r A_1B to ray/beam AB. Both ray/beams AB and A_1B_1 are normal to cut A_1B. Therefore the latter can be considered as segment element, which lies on certain fixed/recorded wave surface. Let further in the image space of cuttings off CC_1 represent the shortest distance between the lattice-type ray/beams CA' and $C_1A'_1$. According to the known theorem

of stereometry, it is possible to assert that both ray/beams CA' and C_1A_1' are normal to the shortest distanc CC_1 between them. Therefore also segment element CC_1 can be considered lying on the fixed/recorded wave surface as standards to which serve ray/beams CA' and $C_1A'_1$. According to the law of tautochronism, the optical length of course of ray between wave surfaces and CC_1 is constant. Therefore $BC = A_1C_1$.

We want so that the image of point A_1 would be point. In that case according to the condition of the formation/education of point image, must be fulfilled the following expression:

$$A_1A_1' = const_2.$$

Here stands to the right the constant, different from constant in expression AA' = const.

Using drawing, it is possible to write

$$A_1C_1 + n'C'A_1' = const_2.$$

Formula AA' = const, also can be presented in the form of the expression

$$nAB + [BC] + n'CA' = const_1. \qquad (\Pi.8)$$

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From point A_1' let us drop perpendicular $A_1'B'$ to ray/beam CA'.

Cut CA' can be presented as difference in cuts CB' and $A'B_1$. Therefore we will obtain

$$nAB+[BC]+n'CB'-n'A'B'=\text{const}_1.$$

Cut CB' there is a distance between the end/leads of the perpendiculars, omitted from ends of the cut C_1A_1' to ray/beam CB'. Therefore cuttings off CB' there is the orthogonal projection of cut C_1A_1' on ray/beam CB' and is expressed by the formula

$$CB'=C_1A_1'\cos d\gamma=C_1A_1'.$$

Here $d\gamma$ is infinitesimal angle between ray/beams CA' and $C_1A'_1$ whose cosine differs from unity by negligible higher order quantity. As a result of the fact that $BC = A_1C_1$ and $CB' = C_1A'_1 \cos d\gamma = C_1A_1'$ expression (II.8) is written thus:

$$nAB+A_1C_1+n'C_1A_1'-n'A'B'=\text{const}_1.$$

Deducting from it the expression

$$A_1C_1+n'C_1A_1'=\text{const}_1,$$

we find

$$nAB-n'A'B'=dC.$$

Here dC is the constant, different from preceding/previous.

From triangles ABA_1 and $A'B'A'$, it is possible to obtain the expressions:

$$AB=dl\cos\alpha;$$

$$A'B'=dl'\cos\alpha'.$$

Because of this let us find from formula $nAB-n'A'B'=dC$, that

$$n\,dl\cos\alpha - n'dl\cos\alpha' = dC.$$

This be cosine law, A. Conradi's for the first time obtained.

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However, in this form this law little befits for practical application/use, in the first place, because in it they are present infinitesimal cuttings off dl and dl' and, in the second place, because in it is the indefinite constant dC.

To remove from this expression infinitesimal values is possible by its term-by-term division on dl. In this case, one should consider that dl'/dl there is linear magnification V in the optical system, a dc/dl - the certain indefinite, but final constant C. Thus, we will obtain the second form of cosine law

$$n\cos\alpha_0 - n'\cos\alpha_0' = C.$$

Eliminating from these expressions value C, we will obtain

$$n(\cos\alpha - \cos\alpha_0) - n'V(\cos\alpha' - \cos\alpha_0') = 0.$$

For imparting to cosine law of more symmetrical form, is solved this equation relatively V:

$$V = \frac{n\cos\alpha - \cos\alpha_0}{n'\cos\alpha' - \cos\alpha_0'}. \qquad (\Pi.9)$$

This expression of cosine law in the most convenient for practical application/use form.

To cosine law, and also and by other, obtained from it in laws, is been inherent one special property which here one should emphasize. First of all let us note that cosine law is fulfilled in such a case, when, substituting in formula (II-9) all possible pairs of the values of angles α and α' within the limits of the focal aperture of this optical system, we from this formula we will obtain always one and the same value of linear magnification in V. For us it is necessary to operate only with ray/beams, which connect points A and A'. But in this case cosine law makes it possible to judge the quality of the image of point A_1, which lies on the other end/lead of the cut dl, aside from course of ray, which connect points A and A'; if cosine law is carried out, then at point A'_1 will be reached the point image of point A_1, otherwise - it will not be.

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Possibility to judge the correction of aberrations at the points, lying on the ray/beams whose course through the optical system is designed, makes it possible to decrease the space of the computational of works, spent during of new optical systems. Although the application/use of electronic computers makes it possible not to

fear complex and tedious computations, the simplification in the work, introduced by cosine law, remains very valuable for a optician-designer.

One should recall that as the prerequisite/premise of the derivation of cosine law serves satisfaction of the condition of formation of point image for one pair of conjugate points A and A_1. The observance of cosine law propagates the accuracy of image to the whole conjugated/combined linear cell/elements dl and dl'. But if the indicated prerequisite/premise in optical system is not carried out, then cosine law loses sense.

Law of cosines easily it is converted into the law of sines with the location of the segment elements dl and dl', shown on drawing (Fig. II.9). The end points A and A' of these cuts (for these points is satisfied the condition of the formation/education of point image) lie/rest on the optical axis of system, and cuttings off themselves dl and dl' are perpendicular to optical axis. In the more general case it is possible to suppose that the ray/beam on which lie/rest these points, does not undergo fracture within system and is at least the axis of the symmetry of the section in question. In order to achieve point image also for the off-axis end/leads A_1 and A_1' of these cuts, is necessary the fulfillment of cosine law.

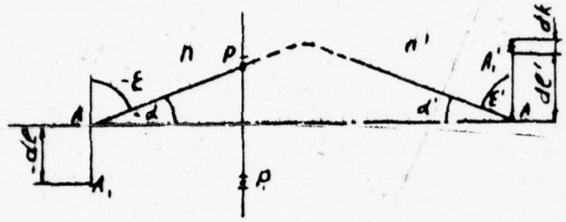

Fig. II.9.

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Since on drawing the angles, formed by ray/beams with cuts dl and dl'_1 are designated in letter ε_1 and, taking into account that the relation dl'/dl in this case really/actually is linear magnification in the optical system, we will obtain

$$V = \frac{n}{n'} \frac{\cos\varepsilon - \cos\varepsilon_0}{\cos\varepsilon' - \cos\varepsilon_0}. \qquad (II.10)$$

From drawing evidently, angles ε and ε' supplement angles α and α' to 90°. Therefore

$$V = \frac{n}{n'} \frac{\sin\alpha - \sin\alpha_0}{\sin\alpha' - \sin\alpha'_0}.$$

For determining angles α_0 and α'_0 let us select initial ray/beam so that it would coincide with the optical axis. Then both angles α and α'_0 become equal to zero, and the expression is simplified.

$$V = \frac{n \sin\alpha}{n' \sin\alpha'}.$$

This is a known formulation of the law of sines, derived E. Abbe. The observance of the law of sines conditions the point image not only of one segment element dl, but whole surface element about by a radius dl, of perpendicular to optical axis system.

C) the condition of the sharp image of segment of curve [3].

Let us examine further the law of the sharp image of more complex object - segment of curve. This law it is convenient to derive/conclude, on the basis of the examination of the common/general/total task of the construction of course of ray in optical system.

Let us suppose that each point of curve $\bar{a}(t)$ sharply is depicted at the appropriate point of curve $\bar{a}'(t)$ (Fig. II.10).

Let $\bar{S}(u, v)$ and $\bar{S}'(u, v)$ designate the directing vectors of the corresponding ray/beams in the object space and images for the fixed value of t.

According to law the farm/truss, optical path length between $\bar{a}(t)$ and $\bar{a}'(t)$ is function only t and does not depend on u and v. We have

$$\bar{s}'\bar{a}'_t - \bar{s}\bar{a}_t = E_\tau .$$

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Let us introduce arc lengths τ and τ_1 curved respectively in the object space and images and an "increase"

$$m = d\tau'/d\tau .$$

Further, let t and t' be the unit vectors, tangential to curves $\bar{a}$ and $\bar{a}'$ Then we have

$$\bar{m}\bar{s}'t' - \bar{s}t = E_\tau .$$

Introducing angles ε and ε' between ray/beams and tangential vectors, we obtain known cosine law

$$\bar{m}\bar{n}'\cos\varepsilon' - n\cos\varepsilon = c_1 .$$

since for the ray/beams, which connect the pair of the corresponding points with curves in the object space and images, value E_τ has one and the same value of c. This value can be determined, if are known angles ε and ε' for any one of the ray/beam, which connects the points in question with curves $\bar{a}$ and $\bar{a}'$.

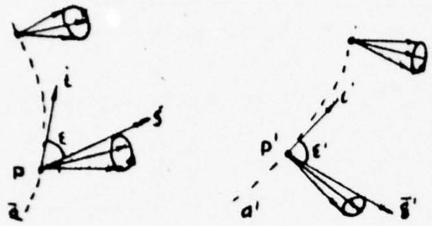

Fig. II.10.

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We obtain

$$\bar{m} n' \cos \varepsilon' - n \cos \varepsilon \doteq \bar{m} n' \cos \varepsilon'_0 - n \cos \varepsilon_0$$

or

$$\bar{m} n' (\cos \varepsilon' - \cos \varepsilon_0) = n (\cos \varepsilon - \cos \varepsilon_0); \quad (\text{П.II})$$

$$\bar{m} = \frac{n}{n'} \frac{\cos \varepsilon - \cos \varepsilon_0}{\cos \varepsilon' - \cos \varepsilon'_0}.$$

Thus, was obtained the same formula which was derived above.

Special interest are of the special cases which follow from the obtained condition:

1) there is a ray/beam, perpendicular both to curved $\bar{a}$, and curve $\bar{a}'$, i.e. $\varepsilon_0 = \varepsilon'_0 = \pi_2$. Then we have

$$m n' \cos \varepsilon' = n \cos \varepsilon. \quad (\text{П.I2})$$

This case, in particular, corresponds to axisymmetric optical and antenna systems whose focal curve is perpendicular to optical axis;

2) there is a ray/beam, tangential to both curves, i.e., $\varepsilon_0 = \varepsilon_0' = 0$. Then

$$mn'\sin^2\left(\frac{\varepsilon'}{2}\right) = n\sin^2\left(\frac{\varepsilon}{2}\right). \qquad (\text{II}.13)$$

To this condition corresponds the optical or antenna system at whose focal curve is parallel or coincides with the axis of system or with the direction of the collimated ray/beams.

The presented methods of optimization as this repeatedly was emphasized that they clear strictly the construction of real optical diagram, i.e., its parameters, which determine the geometry of the reflecting or refracting surfaces. The satisfaction of all conditions - sines, cosines and of so forth only guarantees, h certain approach/approximation, the absence of the determined form of aberration, in any way without affecting the value of other aberrations.

§ 3. Selection of the function of quality.

Let us examine at first problem in general view.

Let us suppose that there is certain system which is determined by the finite number of parameters.

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On the selection of the concrete/specific/actual parameters, depend the properties of system. If the properties of system can be characterized by one number (for this combination of parameters), that correspond to certain function which characterizes system in the function of its parameters, then this function can be call/named estimator, the cost function or qualities.

To optimum system corresponds the minimum either the maximum or the minimax of estimator.

Let certain system have parameters $x_1, x_2, ..., x_n$, and its quality is determined by the function of quality $\Phi = \Phi(x_1, ..., x_n)$. The search for the minimum of function Φ always begins from certain initial vector $x^0 = (x_1^0, ..., x_n^0)$ or of the point n of-dimensional space R_n. The parameters of system are the coordinates of this point [7].

Let us consider that the function of quality ϕ is connected with certain another function u(t, x), determined for points x of space R_n and of points t of space S. The value of function u for the pair of points (t, x) let us call discrepancy. Then to the function of quality corresponds the square of root-mean-square value discrepancy u in region S

$$\Phi(x) = \int_S q(t)\, u^2(t, x)\, dt. \qquad (\Pi.14)$$

Integral is taken on region S . Hearth dt is implied volume element; q(t) - the assigned weighting function. It is obvious, the minimum of integral corresponds to the minimum of the function of quality.

In the case of the linearity of system or with the sufficiently frequent fragmentation of the interval of the variation in the variables it is possible to assume that during transition from point x^0 from function $U^0 = U^0(t, x^0)$ to the new point x the function of quality can be found as

$$\Phi = \int_S (U^0 + \delta U)^2 dt = \int_S \left(U^0 + \sum_{i=1}^{n} \frac{\partial U}{\partial x_i}\delta x_i\right)^2 dt.$$

In this case let us assume that the functions have continuous derivatives in terms of all their arguments and that the corrections are sufficiently small.

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For simplification in the recording, let us assume

$$\frac{\partial U}{\partial x_i} = U_i^0.$$

Then the task of the minimization of the function of quality can be brought to the selection of such parameters x for which the corrections $\delta x_1, \ldots, \delta x_n$ (or vector δx) will be such, that the module/modulus of the vector

$$U(x^0 + \delta x) = U^0 + \Sigma U_i^0 \delta x_i$$

will prove to be minimum. This confirmation makes following sense: at the optimum or close to it state of system, the value of discrepancy U, obviously, will be either minimum (optimum system) or by almost minimum about a small change in parameters x (close to the optimum system).

Corrections δx_i must satisfy normal system of equations

$$\Sigma a_{ji} \delta x_i + b_j = 0 \qquad (j = 1, 2, \ldots, n),$$

where

$$a_{ji} = (U_j^0, U_i^0) \qquad b_j = (U^0, U_j).$$

Here in the parentheses stand scalar products.

The solution of the system of normal equations can lead to the large values of corrections, which contradicts the made assumption

about the linear character of system. It should be noted that the physical sense of corrections δx consists of following: this be nothing else but increments in parameters x of the systems which are converted it of one state into another. In view of the available in practice nonlinearity of the dependence of aberrations on parameters x, the value δx_i they can have different values depending on absolute magnitude of vector x.

During the construction of the algorithm of the minimization of the function of quality, it is important to consider the different role of the separate parameters in the process of minimization.

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Namely, within the limits of each stage it is possible to isolate the most effective parameter whose adjustment makes it possible to maximally decrease the module/modulus of vector U.

After the input/introduction of correction δx_j we obtain the new value of discrepancy, for which is correct the equality

$$|U| = \sqrt{|U^0|^2 - |U_j \delta x_j|^2}.$$

It is obvious, the minimum value U corresponds to maximum $U_j^0 \delta x_j$. Furthermore,

$$|U_j^0 \delta x_j| = \left|U_j^0 \frac{U_i^0 U_j^0}{|U_j|^2}\right| = \left|\left(U_i^0 \frac{U_j^0}{|U_j^0|}\right)\right|,$$

therefore most effective will be the parameter x_j ,for which value $\left|\left(U_i^0 \frac{U_j^0}{|U_j^0|}\right)\right|$ it is maximum. Let us designate this parameter by x_{j_1} or simply x_1. If we are restricted to the addition only of this parameter, then correction $\delta x_{j_1} = \delta x_1$ it will be equal to

$$\delta x_1 = \frac{-(U_j^0 U_1^0)}{|U_1^0|^2}.$$

If the obtained value δx_1 is great, then the entire group of correcting parameters consists of one parameter x_1 and the adjustment of the function of quality is conducted : the direction

$$\delta x = (x_1, \ldots, 0).$$

But if $\delta x \leq \eta$ (where η is the selected value), then one should attempt to expand the group of the amended parameters, after supplementing parameter x_1 even by any parameter x_j . So that this parameter would be most effective of all that which were remaining the scalar product of the changed after the introduction of correction vector U^0 for vector $U_{j_2}^0$

$$\left(U^0 + U_1 \delta x_1, \frac{U_{j_2}}{|U_{j_2}|}\right)$$

must be maximum on module/modulus.

Let us assume that $\delta x_{j_2} = \delta x_2$. Then as the effective pair of the

parameters we accept parameters 1 and 2 and compute corrections δx_1 and δx_2 (remaining parameters x_3, x_4, ..., x_n are fix/recorded), for which the module/modulus of vector will be smallest.

End section.

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If corrections are great ($\max|\delta x_i| > \eta$) then the entire group of the parameters must consist of x_1 and x_2. Otherwise it is possible to supplement the group of the amended parameters by parameter x_3 so forth. Thus, can be found the group of the parameters $x_1, x_2, \ldots, x_\ell$ and vector δx with coordinates $(\delta x_1, \delta x_2, \ldots, \delta x_\ell)$, that possessing the following property:

1) its maximum in module/modulus coordinate does not exceed the selected value η ;

2) the maximum coordinate of the vector δx of the expanded group cf parameters $x_1, x_2, \ldots, x_{\ell+1}$ will be more than η .

The process of the construction of the vector of direction δx it is virtually reduced to the orthogonalization of the system of vectors $\{U_i\}$.In this case, the search for direction δx will be considered final, if after the input/introduction of next ℓ - that coordinate has the inequality

$$(U^0, U_\ell) > K_1 (U^0 + \sum_1^\ell \delta x_i U_i U_{\ell+1}),$$

where k is the assigned number.

In the obtained direction δx, is realized the search for the minimum value of function of quality Φ .

For the preservation/retention/maintaining of the generality of presentation, it should be noted that the position concerning the effective parameter is correct upon consideration of the limitations which must be superimposed to parameter values. In this case it is possible, as earlier, to isolate the initial stage of the optimization when for the extent/elongation of a small series of large increments in the effective parameters it is possible to substantially decrease U and then begins the prolonged process of the gradual decrease U because of the remaining, less effective parameters.

Let us examine now in more detail physical nature discrepancy U in connection with the task of the optimization of the optical system which composes the final gcal of present section.

For an optical system as the function of quality, it is

convenient to select the RMS value of aberrations either on focal surface or in aperture, i.e., wave aberrations.

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For example, expression for the function of quality can take the following form:

$$\Phi = \sum_{j=1}^{m} (\Delta q_j^2 + G_j^2) q_j + m\kappa_f \Delta f^2 + m\kappa_M \Delta M^2, \quad (\Pi.15)$$

where Δq_j are aberrations in meridian cut;

G_j - Aberration in sagittal section;

m - a quantity of ray/beams for which are determined aberration;

$q_j \kappa_f \kappa_M$ - weight coefficients.

The relationship/ratios:

$$\Delta f = f - f^0;$$
$$\Delta M = M - M^0$$

is determined the standard deviation of focal length f and of apical cut m of the assigned magnitudes f^0 and M^0.

Thus, as discrepancy during the optimization of optical systems we will utilize aberrations, and among the latter preference will be

given up to wave aberrations.

In this case, it seems to us that during the analysis of optical-type antennas it is possible to use three in principle equivalent methods of evaluating the distortions:

1) in the range of parallel beam;

2) in the interval/gap between the reflecting (refracting) surfaces;

3) in the region of emitter.

In the first case is examined the operating mode in transmission and are rate/estimated distortions according to the deviation of real wave front from certain plane (Fig. II.11).

In the third case is examined the work to reception/procedure and distortions are rate/estimated according to the deviation of real wave front from certain standard sphere (Fig. II.12).

The second case corresponds to the mixed mode/conditions; it is assumed that on the main mirror of two-mirror antenna falls the parallel beam, inclined on certain angle (reception/procedure), and on auxiliary mirror (II.13a) falls spherical front from certain outlying source (transmission).

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DESIGN OF OPTIMUM TWO-MIRROR ANTENNAS WITH THE OSCILLATION OF R--ETC(U)

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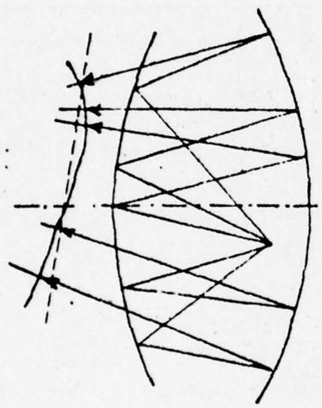

Fig. II.11.

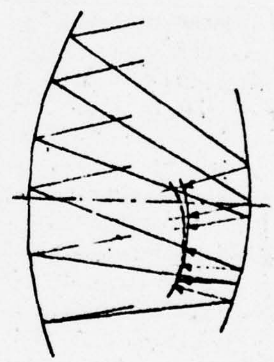

Fig. II.12

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It is obvious, if both fronts reflected coincide (Fig. II.13a, b), then distortions are absent; otherwise the degree of a difference in these fronts is proportional to the aberrations of system. In

specific cases the preference can be returned to one or the other case.

1. Let us examine first case. Let in Fig. II.14 on auxiliary mirror fall the light beam from the source, outlying from focus into the point, characterized by vector $\bar{P}$. Then the reflected beam can be found from the vector relationship/ratio

$$\bar{y}_1 = (C_1 - |\bar{R}_1 - \bar{\rho}|)\,\bar{\xi}_1 + \bar{R}_1, \qquad (\text{П.I6})$$

where $\bar{R}_1$ is the vector, which determines the surface of auxiliary mirror. In the general case it is assigned by the differential equation

$$\frac{dR_1}{d\varphi} = \overset{\circ}{R}_1 \frac{f\sin\varphi + 2\,\mathrm{tg}\frac{\varphi}{2}(d - R_1)}{2\left(d - f\sin^2\frac{\varphi}{2}\right)}; \qquad (\text{П.I7})$$

$\bar{\xi}_1$ - single standard to wave front

$$\bar{\xi}_1 = \bar{i}_1 - 2\bar{\eta}_1(\bar{\eta}_1\bar{i}_1). \qquad (\text{П.I8})$$

reflected.

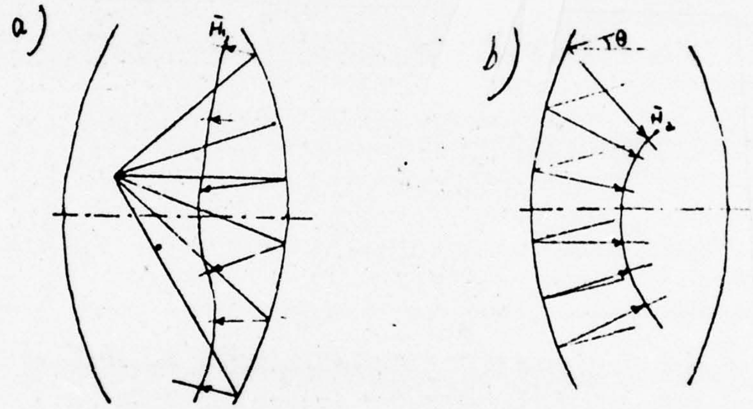

Fig. II.13.

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Here $\bar{i}_1$ is the unit vector, directed along incident ray: $\bar{\eta}_1$ is a standard to reflector.

For the interpretation of equation (II.18) let us introduce the

vector of source $\bar{P}$, which determines according to value and direction the deflection of irradiator from focus (Fig. II. 14). Here the position of ray/beams in incident beam is determined by angles with X-axis and y axis. Then in the meridian plane of the component of the vector, directed along the incident ray:

$$\cos_x = \frac{R_1\cos\varphi - x_1}{\sqrt{(R_1\cos\varphi - x_1)^2 + (R_1\sin\varphi - y_1)^2}};$$

$$\cos_y = \frac{R_1\sin\varphi - y_1}{\sqrt{(R_1\cos\varphi - x_1)^2 + (R_1\sin\varphi - y_1)^2}}.$$

In order to assign indirect wave, we will use Fig. II.15. From the figure one can see that the current point on auxiliary mirror is assigned by radius $R_1 \sin \phi$ and by an angle α

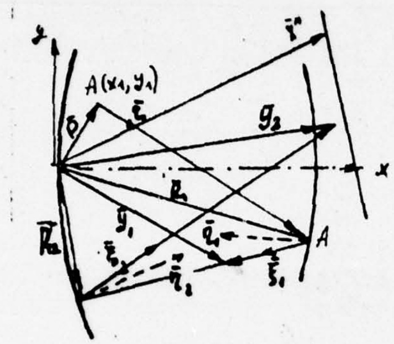

Fig. II.14.

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Then the components of vector $\bar{i}_1$ are equal to:

$$\left.\begin{aligned}
\cos_x &= \frac{R_1\cos\varphi - x_1}{\sqrt{(R_1\cos\varphi - x_1)^2 + (R_1\sin\varphi\cos\alpha - y_1)^2 + (R_1\sin\varphi\sin\alpha)^2}};\\
\cos_y &= \frac{R_1\sin\varphi\cos\alpha - y_1}{\sqrt{(R_1\cos\varphi - x_1)^2 + (R_1\sin\varphi\cos\alpha - y_1)^2 + (R_1\sin\varphi\sin\alpha)^2}};\\
\cos_z &= \frac{R_1\sin\varphi\sin\alpha}{\sqrt{(R_1\cos\varphi - x_1)^2 + (R_1\sin\varphi\cos\alpha - y_1)^2 + (R_1\sin\varphi\sin\alpha)^2}}.
\end{aligned}\right\} \quad (\text{II.19})$$

Parameters of standard $\bar{\eta}_1$ to the auxiliary mirror:

$$\begin{aligned} \cos_x &= \cos(\varphi-\rho); \\ \cos_y &= \cos\alpha \sin(\varphi-\rho); \\ \cos_z &= \sin\alpha \sin(\varphi-\rho). \end{aligned} \qquad (\text{II}.20)$$

Here angle ρ is determined by the differential equation

$$\frac{1}{R_1}\frac{dR_1}{d\varphi} = \operatorname{tg}\rho.$$

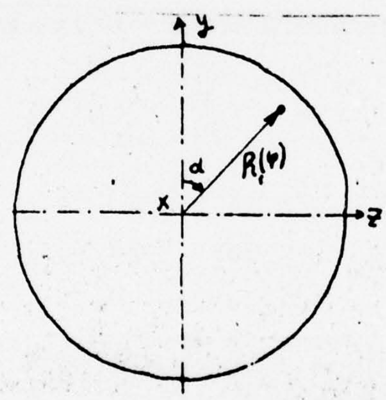

Fig. II. 15

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Thus, are obtained all cell/elements of relationship/ratio (II.16). As concerns value of C_1, which determines path from the falling/incident front to that which was reflected, then it, obviously, must be more distance from point with coordinates x_1 y_1 of the most removed edge of auxiliary mirror (Fig. II.14). The vector of

the falling/incident front $\bar{x}_1$ can be assigned directly at the point of the location of source. Then the difference

$$|\bar{R}_1-\bar{\rho}| = \sqrt{(R_1\cos\varphi - x_1)^2+(R_1\sin\varphi\cos\alpha - y_1)^2+(R_1\sin\varphi\sin\alpha)^2}. \quad (\text{II}.21)$$

Let us find now the parameters of the front, reflected from the main mirror

$$\bar{y}_2 = (C_2 - |\bar{R}_2 - \bar{y}_1|)\,\bar{\xi}_{12} + \bar{R}_2. \qquad (\text{II}.22)$$

Equality (II.22) it is recorded taking into account the fact that here falling/incident front $\bar{x}_2$ is equal to the front, reflected from the auxiliary mirror $\bar{y}_1$.

The equation of the main mirror $\bar{R}_2$ is determined in the parametric form

$$|\bar{R}_2| = \sqrt{(f\sin\varphi)^2+\left\{\frac{4d(R_1-d)+f\sin^2\varphi(f-2R_1)}{2[2d-R_1(1-\cos\varphi)]}\right\}^2}, \quad (\text{II}.23)$$

where R_1 is a radius-vector of auxiliary mirror.

For determining entering here R_1 and ϕ it is necessary to find the coordinates of the point of intersection of the ray/beam of front $\bar{y}_1$ reflected from auxiliary mirror, with main mirror. This means that it is necessary to solve the system of equations of the meeting of the ray/beam

$$\frac{x-\bar{y}_{1x}}{\xi_{1x}} = \frac{y-\bar{y}_{1y}}{\xi_{1y}} = \frac{z-\bar{y}_{1z}}{\xi_{1z}}$$

and of the surface

$$|\bar{R}_2| = \sqrt{(f\sin\varphi)^2 + \left[\frac{4d(R_1 - d) + f\sin^2\varphi\,(f - 2R_1)}{2[2d - R_1(1-\cos\varphi)]}\right.} \;;$$

$$\frac{1}{R_1}\,\frac{dR_1}{d\varphi} = \frac{f\sin\varphi + 2\,\mathrm{tg}\frac{\varphi}{2}\,(d - R_1)}{2(d - f\sin^2\frac{\varphi}{2}}.$$

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Here $\bar{y}_{1x}$, $\bar{y}_{1y}$, $\bar{y}_{1z}$ - the coordinate of point on the ray/beam, reflected from auxiliary mirror, $\bar{\xi}_{1x}$, $\bar{\xi}_{1y}$, $\bar{\xi}_{1z}$ - the component standards, directed along this ray/beam.

System of equations is solved by the method of iterations.

The unit vector, normal to wave front $\bar{y}_2$ reflected, will be

$$\bar{\xi}_2 = \bar{\xi}_1 - 2\bar{\eta}_2(\bar{\eta}_2\bar{\imath}_2).$$

Here the components of standard $\bar{\eta}_2$ to the main mirror:

$$\left.\begin{aligned} \cos_x &= -\cos\gamma; \\ \cos_y &= \cos\alpha_1 \sin\gamma; \\ \cos_z &= \sin\alpha_1 \sin\gamma, \end{aligned}\right\} \quad (\text{II}.24)$$

where

$$\operatorname{tg}\gamma = \frac{(f - R_1)\sin\varphi}{2d - R_1(1-\cos\varphi)}$$

The components of standard $\tilde{\xi}_1$ to wave front of the incident can be found after the substitution of values (II.20) and (II.21) in (II.18).

Constants c_1 and c_2 determine position front, that left the antenna, and they are assigned from condition, for example, that the last/latter wave front intersects aperture.

By the essential torque/moment of the calculation of aberrations is the process of the comparison of the front y_2 reflected about the standard plane $\tilde{x}^*$. Essence consists of following: let Fig. II.16 be depicts the wave front $\bar{y}_2$ and plane $\tilde{x}^*$. In order to rate/estimate aberrations in antennas, obviously, necessary in a some manner to deduct one front of another and to find the RMS value of the obtained

differences.

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However, how to find this difference and which points of both of fronts to consider mutually appropriate? It is obvious, there is as much as desired methods in order to establish/install this conformity; as an example can serve figure II.16a,c. It is here shown two cases. In the first are compared the points with identical ordinate, in second point, which lie on standards to plane of reference. Apparently, it is not possible to give categorical preference to any comparison method, but we will compare both fronts with respect to standards to the front $\bar{y}_1$ reflected (Fig. II.16c). In this case, it is necessary to find the points of intersection of straight line, characterized by direction $\bar{\xi}_2$, and to plane $\bar{x}^*$, and then the point, which lies on the terminus of vector $\bar{y}_2$, and finally to determine the distance between them. Analytically this means that it is necessary to find the intersection of straight line

$$\frac{x-\bar{y}_{2x}}{\xi_{2x}}=\frac{y-\bar{y}_{2y}}{\xi_{2y}}=\frac{z-\bar{y}_{2z}}{\xi_{2z}} \qquad (II.25)$$

with plane characterized by the equation

$$\ell(x-x_0)+m(y-y_0)+n(z-z_0)=0. \qquad (II.26)$$

Here

$$\ell=\cos\theta;\ m=\sin\theta;\ n=1,$$
$$x_0=d;\ y_0=0;\ z_0=0.$$

i.e. this can be the plane, passing through the apex/vertex of auxiliary mirror at an angle θ to the vertical y axis. Let us call/name conditionally the coordinates, obtained during joint solution equation (II.25) and (II.26) letters x_x^*, x_y^*, x_z^*. Then the deviation of front $\bar{y}_2$ from front $\bar{x}^*$ at the particular point will be

$$\Delta L = \sqrt{(\bar{y}_{2x} - \bar{x}_x^*)^2 + (\bar{y}_{2y} - \bar{x}_y^*)^2 + (\bar{y}_{2z} - \bar{x}_z^*)^2}. \quad (\text{II}.27)$$

To evaluate common/general/total deviation, we will use integral of standard deviation with respect to entire aperture, characterized by the angles φ_1 and φ_2

$$Q = \int_{\varphi_1}^{\varphi_2} \Delta L^2 \, d\varphi. \quad (\text{II}.28)$$

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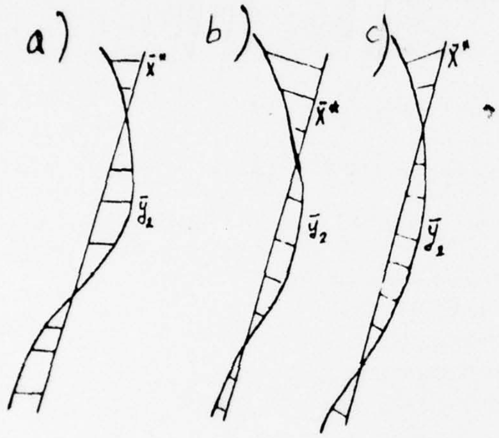

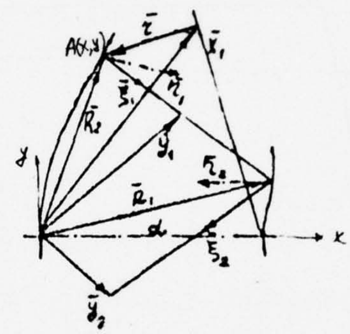

Fig. II.17

Fig. II.16.

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This equation is that function of quality which is utilized in the process of optimization and optimum synthesis.

By P. Rassmotry the second case. In this case, let us turn to diagram in Fig. II.17. Here on main mirror falls light beam at an angle θ to x-axis. Then the parameters of standard to the falling/incident front:

$$\left.\begin{aligned} \bar{i}_x &= \cos\theta; \\ \bar{i}_y &= \sin\theta; \\ \bar{i}_z &= 1. \end{aligned}\right\} \qquad (\text{II}.29)$$

The parameters of standard to main mirror are determined by equations (II.24), and the surface of the main thing by mirror-equations (II.23). Substituting (II.23), (II.24), (II.29) into equation of the type (II.16), let us find the front, reflected from the main mirror when not it falls the flat/plane inclined front:

$$\bar{y}_1 = (c_1 - |\bar{R}_2 - \bar{x}_1|\,\bar{\xi}_1 + \bar{R}_2;$$
$$\bar{\xi}_1 = \bar{i} - 2\bar{\eta}_1(\bar{\eta}_1, \bar{i}).$$

The equation of the falling/incident front, or more precisely, difference $|\bar{R}_2 - x_1|$, let us find, after using Fig. II.17. Here the plane of the falling/incident front is assigned by the equation

$$x\cos\theta + y\sin\theta - d\cos\theta = 0.$$

Then distance from point x_A, y_A of this plane

$$|\bar{R}_2 - \bar{r}_1| = |(x_A \cos\theta + y_A \sin\theta - d\cos\theta)|.$$

By selection C_1 we attain that the front $\bar{y}_1$ being investigated would render/show in the interval/gap between mirrors. The front, reflected from auxiliary mirror and proceeding from the source, outlying from focus into point with coordinates x_1, y_1, already we have found and it is expressed by equations (II.16)-(II.23).

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Obviously, if front $\bar{y}_1$ and front (II.16)-(II.3) during the proper selection of constants C_1 coincide, then of distortions in system they are absent; otherwise the degree of the disagreement of these fronts will determine the value of distortions in system.

III. the third case corresponds to an incidence/drop in the flat/plane inclined front on antenna; he is studied the deviation of the front, reflected by auxiliary mirror, from sphere. From main mirror is reflected front $\bar{y}_1$, and then it is converted into front $\bar{y}_2$ after reflection from auxiliary mirror. The investigation of aberrations in this case consists in the computation of the distance from point x_1, y_1 with alternating/variable position of front $\bar{y}_2$

$$L = \sqrt{(y_{2y} - y_1)^2 + (y_{2x} - x_1)^2},$$

which then is compared with certain supporting/reference distance, i.e., is determined standard deviation ΔL. The values of the components of this relationship/ratio were already found in the preceding/previous cases.

§4. Formulation of the problem of the synthesis of the optimum scanning optical-type antennas.

Cne Of the most urgent tasks the antenna cf technology still is the task of constructing of the wide-angle scanning antennas (mirrors and lenses).

Despite the fact that as such antennas are applied long-focus parabolic reflectors, aplanatic and bifocal two-mirror and lens antennas and the tasks of scanning to a certain degree are solved, it seems to us that this solution in the majority cf cases far from the cptimum.

Actually, the requirements which before(are to the scanning antennas, it is possible to systematize approximately as follows:

1) is assigned the sector of continuous scanning;

2) are assigned the angles of the discrete location of ray/beams;

3) is assigned focal curve and sector of scanning.

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It is obvious, in all cases it is assumed that the distortions of radiation pattern must be minimum or, in the extreme case, equal to zero for all beam directions.

Without doubt are solved the placed problems and which antennas are utilized? Besides spherical antennas and Luneberg's lenses which invariants relative to the direction of scanning, most widely are utilized the indicated antennas (parabolic and aplanatic). However, such antennas are palliative by their the very nature: they are designed for obtaining ideal ray/beam only at one point (paraboloid, aplanat) or at two points (cylindrical bifocal antenna with line sources), and during the deviation of source from these points (foci) occurs a progressive increase in the distortions (Fig. II.18). As concerns the focal curve in which are arrange/located the sources, then it, obviously, must be only such, which has this antenna.

It seems to us that much more adequate to stated problem must be the antennas, specially designed for each case, i.e., satisfying the placed condition in the assigned best approximation. In this setting this to problem is reduced to problem of the synthesis of certain equipment/device in accordance with the assigned requirements.

We examined above the task of optimization; its essence consisted in the fact that among the assigned type of antennas (for example, aplanatic) and with were placed conditions it was selected such version which during the concrete/specific/actual combination of its parameters in a best (in known sense) manner satisfies the stated requirements. However, the type of antenna from this is not changed, since the solution of problem was searched for in just one class - the form of characteristic curve was not subject to change.

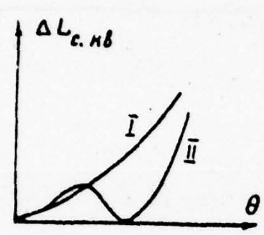

Fig. II.18. I - monofocal antenna. II - bifocal antenna.

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Unlike the procedure of optimization, synthesis, is provided for first of all precisely a variation of characteristic curve, including ay the same time simultaneous optimization as part of the process. Therefore in the final analysis for each specific case of use, can be constructed completely specific antenna.

The problem of the synthesis of optimum antennas to a certain degree is analogous to the tasks of the construction of optimum cybernetic systems, in connection with which as the basis of antenna synthesis can be placed analogous matemathical apparatus. In this case, without damage for generality, we can to assume that in our

case occurs the dynamic system (antenna), controlled by digital computer. In this case, let us examine the systems which are assigned by the fundamental differential equation

$$\frac{dr}{d\varphi} = F_1\left[\varphi, r, f(\varphi), dM\right] \qquad (\Pi.30)$$

and the series parametric equations:

$$\left.\begin{aligned} x &= F_2\left[\varphi, r, \frac{dr}{d\varphi}, f(\varphi), M\right]; \\ y &= f_1(\varphi)\sin\varphi; \\ \alpha &= f_2(y). \end{aligned}\right\} \qquad (\Pi.31)$$

Analytically our task can be formulated as follows. For certain of the dynamic system, described by equations (II.30), (II.31) and characterized by the index of the quality

$$\Delta L\left\{\left[\frac{dr}{d\varphi}, r, f(\varphi), \ldots, \theta\right] - x^*(\varphi)\right\},$$

to ensure the minimum, the maximum or finally the minimax of certain integral

$$Q = \int_{\varphi_1}^{\varphi_2} \Delta L^2 \, d\varphi, \qquad (\Pi.32)$$

which is functional from some steering functions:

$$\left.\begin{aligned} y_y &= f_1(\varphi)\sin\varphi; \\ \alpha &= f_2(y), \\ x_1 &= f_3(y_1). \end{aligned}\right\} \qquad (\Pi.33)$$

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The selection of functions (II.33) is limited, i.e., $y \in Y$, $x \in X$, $\alpha \in A$, a the extremum of functional is searched for in certain **permissible** interval of the angles

$$\varphi_1 \leq \varphi \leq \varphi_2 .$$

I. Simplest task of optimum synthesis includes creation of antenna for which must be provided minimum cf integral

$$Q = \int_{\varphi_1}^{\varphi_2} \Delta L^2 \, d\varphi \qquad \text{(II.34)}$$

for assigned angle of deflection of radiaticn pattern θ. Then the conditions of task can be formulated as follows: to find function $f_1(\varphi)$, which it would supply the minimum to functicnal (II.34) for the dynamic system:

$$\left.\begin{aligned} &\frac{dr}{d\varphi} = F_1\left[r, f_1(\varphi), \ldots, \varphi\right]; \\ &x = F_2\left[r, \frac{dr}{d\varphi}, \ldots, \varphi\right]; \\ &y = f_1(\varphi)\sin\varphi; \\ &\alpha = f_2(y) \end{aligned}\right\} \qquad \text{(II.35)}$$

with the limitations:

$$\left.\begin{aligned} d_2 \leq d \leq d_1; \\ M_2 \leq M \leq M_1; \\ \varphi_2 \leq \varphi \leq \varphi_1; \end{aligned}\right\} \qquad (\text{II}.36)$$

$$\alpha = \alpha(\varphi) = 0. \qquad (\text{II}.37)$$

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Here, as can be seen from (II.37), the discussion concerns monofocal antenna with focus on the axis.

II. Second task of optimum synthesis can be considered as variety of the first: its condition can be propagated to bifocal antenna, after replacing equation (II.37) with equation

$$\alpha = \alpha(\varphi) = \pm A.$$

III. Third task is minimization of functional (II.34) for sector $\pm\theta$ with assigned trajectory of movement of source. It is obvious, this task to a certain degree can be solved for mono- or bifocal antenna. Then limitations take the form

$$
\begin{aligned}
& d_2 \leq d \leq d_1; \\
& M_2 \leq M \leq M_1; \\
& \varphi_2 \leq \varphi \leq \varphi_1; \\
& \alpha = \alpha(\psi) = 0 \quad [\alpha = \alpha(\psi) = \pm A]; \\
& x_1 = F_2(y_1).
\end{aligned}
$$

IV. The preceding/previous task can have interesting solution, if we are not limited to the antenna, which has focus (or two foci), namely, the fourth task of optimum synthesis has the following formulation: in sector $\pm\theta$, the antenna must have minimum error (for example, constant). This task to us is represented most adequate for antennas with wide-angle scanning. Let us formulate it in the terms of variational methods.

Let the sector of scanning be broken into N of intervals. Then for each interval it is possible to record integral (II.34):

$$
\left.\begin{aligned}
& Q\Big|_{\frac{\theta}{N}} = \int_{\varphi_1}^{\varphi_2} \Delta L^2 \, d\varphi; \\
& Q\Big|_{\frac{2\theta}{N}} = \int_{\varphi_1}^{\varphi_2} \Delta L^2 \, d\varphi; \\
& \cdots\cdots\cdots \\
& Q\Big|_{\theta} = \int_{\varphi_1}^{\varphi_2} \Delta L^2 \, d\varphi.
\end{aligned}\right\} \qquad (\text{II}.38)
$$

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Here limitations are analogous systems (II.36), and as the minimizing functions they are utilized:

$$y = f_1(\varphi)\sin\varphi;$$
$$x_1 = f_2(y_1);$$
$$\alpha = f_3(\varphi).$$

V. As the following task of optimum synthesis, it is possible to examine the method of the construction of antennas, similar bifocal. As is known, bifocal antennas are designed only for one section, which contains axis of antenna and both foci. Aberrations in remaining sections (astigmatism) in any way are not monitored; in connection with this to rationally assign more common/general/total

mission. Namely, to find these two control functions:

$$\alpha = \alpha(\varphi);$$
$$y = f_1(\varphi)\sin\varphi,$$

which would give the minimum to functional (II.34) for values $\pm\theta$. With this precise convergence in meridianal section (as the bifocal antennas) will not have. But aberrations as a whole will be decreased because of a compromise between aberrations in different planes.

The formulated in present paragraph tasks of optimum synthesis, apparently, do not exhaust all possible situations which can arise, during the development of the scanning antennas. However, these full-universal, and, obviously, in equal measure they can be solved not only in connection with two-mirror antennas, but also to the antennas lens, mirror-lens, or to antennas with point source.

It should be noted that the setting of the variational problem of the synthesis of optimum antennas first becomes possible only in view of the essential progress of computer technology, and also the appearance of new investigations in the field of logic and algorithms of the solution of the nonlinear problems of the optimum synthesis: dynamic programming and the principle of maximum [8, 9, 11-15].

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Virtually the essence of the problem of the creation of the optimum scanning antennas consists precisely of the development of this logic of machine calculation so that even at the existing enormous velocities TsVM task could be solved for the foreseeable interval of machine time.

§5. Construction of the elementary operation of synthesis according to approximation method in the space of strategies.

Let us examine the diagram of the methcd cf the optimum synthesis cf two-mirror antenna, based on approximation in the space of strategy [8.14].

As the basis of method, can be placed the following principle of needle-shaped variations: for some arbitrary or obtained by other previously methods characteristic curve is assumed that its small section can change within certain limits by low value. In this case, entire/all remaining part of characteristic curve remains constant/invariable; however, varying one region of curve, even very small, we nevertheless each time we obtain, strictly speaking, new antenna to new aberrations. From all variations in the section of characteristic curve in question is selected that, that provides the

minimum of functional, aberrations, designed for an entire antenna.

Fundamental requirement for needle-shaped variations consists of the preservation/retention/maintaining of the continuity of characteristic curve. As is known, function $f(x)$ he is called continuous, if to the small change x corresponds a small change in the function $f(x)$.

Last/latter determination needs the refinement: it is possible to consider the close of function y(x) and $y_1(x)$ in such a case, when the module/modulus of their difference

$$y(x) - y_1(x)$$

is small for all values of x, for which are assigned functions y(x) and $y_1(x)$, i.e., to consider close curved, close in ordinates.

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However, in many instances are more logical to consider close only those curves, which are close in ordinates and in the directions of tangents at corresponding points, i.e., for close curves not only small the module/modulus of difference $y(x)-y_1(x)$, but is also small the module/modulus of difference $y'(x)-y'_1(x)$.

Therefore it is possible to count that the curves of $y = y(x)$

and $y = y_1(x)$ are close in the sense of zero-order nearness, if the module/modulus of difference $y(x)-y_1(x)$ ml. Furthermore, curved $y_1(x)$ and $y(x)$ are close in the sense of first-order nearness, if the module/moduli of differences $y(x)-y_1(x)$ and $y^1(x)-y^1_1(x)$ are small.

So, Fig. II.19 depicts curves, close in the sense of zero-order nearness, but close in first-order sense, since ordinates in them are close, and the directions of tangents are not close.

The curves of Fig. II.20 are close in the sense of nearness of the I order. The requirement for the continuity of characteristic curve can be formulated as follows: the function $y = f(\varphi)\sin\varphi$

is continuous when $\phi = \phi_0$, if for any ε it is possible to fit $\delta > 0$ such, that

$$|y(\varphi) - y(\varphi_0)| < \varepsilon$$

with

$$|\varphi - \varphi_0| < \delta.$$

The concept of extremum with a variation in the individual parts of characteristic curve also needs refinement.

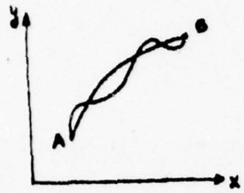

Fig. II.19.

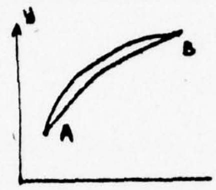

Fig. II. 20

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Speaking about the minimum, we will bear in mind the smallest value of functional only with respect to the values of functional in close curves. But, as it was shown that nearness of curves can be understood differently; therefore in the determination of the maximum or minimum, we will distinguish the degree of this nearness.

If the functional of phase error reaches in the curve of $y = y_0(\phi)$ the minimum according to relation to all curves, for which the module/modulus of difference $y(\phi)-y_0(\phi)$ is small, i.e., with respect

to curved, close to $y = y_0(\phi)$ in the sense of zero-order nearness, then minimum it will be weak. But if functional reaches in the curve of $y = y_0(\phi)$ the minimum only with respect to curves, close to $y = y_0(\phi)$ in the sense of first-order nearness, i.e., with respect to curved, close to $y = y_0(\phi)$ not only in ordinates, but also in the directions of tangents, then the minimum will be powerful.

Apparently, in the first stages of synthesis it is possible to assume that the target/purpose of each needle-shaped variation on this section of characteristic curve will be obtaining the weak minimum of the functional, which determines phase error.

Now, when is given definition of needle-shaped variation as basic operation of antenna synthesis, let us show, as it can be realized irrespectively of the specific problem of synthesis.

Let there be certain monofocal two-mirror antenna, presented in Fig. II.21 and II.22.

It is further necessary to find the characteristic function which would provide minimum rms error and satisfied some requirements whose formulation to us is not now important.

During the solution of problem as approximation method in space

- strategies let us accept for base N - the line-of-communication process, in each stage of which is varied certain region of characteristic curve (needle-shaped variations). So, by Fig. II.21 it is accepted that is varied the region from point A to point B(B^1, B_2...), moreover point A is constant/invariable, and point B consecutively accepts a series of positions. The new sections AB must, obviously, to be tangents to the remaining part of characteristic curve at point A.

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Then during the first stage of the first stage we are have an antenna from characteristic curve $B^1AOA_1B^1{}_1$; at the second stage - antenna from characteristic curve $B^2AOA_1B^2{}_1$ so forth. For each of these curves, is calculated the functional of quadratic phase error and is selected curve, that ensures minimum value.

In this case, the results of calculation are compared with the value of phase error, obtained for an antenna from initial characteristic curve with the optimum for it location of source.

At each stage of the next stage of synthesis, we deal only with the value of functional, minimum for the preceding/previous stage and with the results of calculations at this stage of this stage, i.e.,

in the memory of machine, always is stored only one value of functional (in the case of one-dimensional control) unlike the radiation synthesis where it is required to store a whole series of the values of functionals, since must be carried out the parallel sorting of the parameters of immediately several ray/beams.

One should note also that on the strength of the discreteness of task we actually utilize not entire curve on section AB (B^1, B^2,...), but only one point B(B^1, B^2,...), more precise, only its abscissa $\ell(\varphi)\cos\varphi$, since ordinate y it corresponds to the intervals of discreteness and is accepted as constant for this stage of optimization. However, with this setting immediately appears the problem of the selection of the permissible positions of point B(B^1, B^2,...).

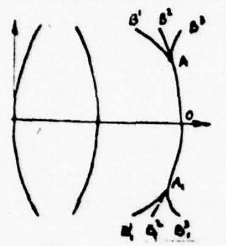

Fig. II.21.

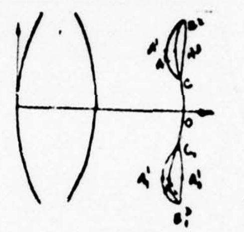

Fig. II.22.

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It is obvious, this point so must be arrange/located in space so that general characteristic curve would not have fractures and sections, parallel the axes of antenna, since antenna from such characteristic curve cannot be realized in practice.

Let us examine the following (the second) stage of synthesis (Fig. II.22). On this stage are record/fixed position of the best point of the preceding/previous stage (for example, point B^3) and the

position of point C, which is subject to variation in the subsequent, 3rd stage of synthesis.

The second stage lies in the fact that is taken a series of the values of the abscissa of point $A(A^1, A^2, \ldots)$ and for obtained characteristic curves (for example, $B^3\ A^3\ CCC_1\ A^3\ B^3{}_1$) is calculated the value of functional for entire antenna aperture. In this case, it can seem that the obtained value of quadratic phase error will be equal, is more or less than the phase error, calculated for characteristic curve B^3 AOA $B^3{}_1$. If not one of the values of functional will be less than preceding/previous, then, obviously, it is necessary to leave the old value of the abscissa of point A, which corresponds to the first stage, and to pass to following stage. On the other hand, it should be noted that in each stage it must be (n + 1) stages, moreover value must be limited only to torque/moment of obtaining the outer limit: on (n + 1)-th stages the result must be worse than at n-th stage. This, it goes without saying, is correct, if with a variation in the coordinates of point A occurs a decrease (or an increase) in the phase error of an entire antenna in comparison with the result of the first stage.

Let in the second stage of synthesis the best result be obtained for characteristic curve $B^3\ A^2\ COC_1\ A^2{}_1\ B^3{}_1$. Then on the third stage of optimum synthesis is detented the position of points $B^3\ A^2$ and of

an entire remaining part of characteristic curve, except point C; the schematic of the process of the optimization of the coordinates of point C the same as in the preceding/previous, 2nd stage; as a result of the sorting of abscissas, is selected the value $f(\varphi)\cos\varphi$ ensuring the minimum of phase error in this stage.

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Figures II-23 shows the two-mirror antenna at whose characteristic curve is subjected to needle-shaped variation in interval $y_A = |y_C|$. It is easy to see that this form change of characteristic curve will lead to the appropriate changes in the airfoil/profiles of main and auxiliary mirrors. This in turn, will cause a change in the direction of the beams, reflected from mirrors.

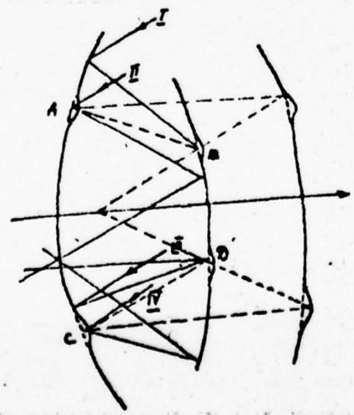

Fig. II-23.

Page 77. Chapter III.

ALGORITHMS OF SYNTHESIS.

By algorithm [15] m we understand certain formal instruction according to which it is possible to obtain the necessary solution of problem. This formulation, is understood it does not pretend to accuracy, but fast expresses that intuitive view to algorithm which was formed even in antiquity. Term "algorithm" proceeds on behalf in the name of the medieval Uzbek mathematician Al'Khorezmi, who even in IX V. gave the rules of the fulfillment of four arithmetic operations in decimal system.

To any algorithm are characteristic some common properties:

I) the determinancy of algorithm. Is required, so that the method of action (computation) would be so precise and generally understandable, so that would not remain the places to the arbitrariness:

2) the mass character of algorithm. Algorithm serves not for the solution to any specific problem, but for the solution of the whole

class of tasks. Indications on the method of action are used to the initial conditions which can be varied;

3) the result of algorithm. This property, called sometimes the directivity of algorithm, it requires so that the algorithmic procedure, used to any task of this type, through the finite number of spaces would be stopped and after stop it would be possible to deduct the unknown result.

Algorithms, in accordance with which to actions, I call numerical algorithms. Numerical are the algorithms, expressed formulas or the diagrams, serving for the solution of certain class of tasks, if by these formulas is completely expressed both the composition of actions and the order in which they must be fulfilled.

Is feasible also the logical algorithm, which can be the independent method of action, and it can enter in the composition of numerical algorithm, for example on the basis of intermediate results, must be accepted solution to the character of the subsequent actions or even to the cessation of further calculations.

However, both in the case of numerical and in the case logical algorithms the content one and the same: in both cases the algorithm is defined as system of rules for the solution of a defined class of

tasks, which possesses the properties of determinancy, mass character, result.

Without doubt must be organized computational operation at each stage of synthesis? This task does not have standard solution with the synthesis of nonlinear systems and dynamic programming gives only the the general procedure of solution elementary operation each time must correspond to the content of specific problem.

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As the special feature/peculiarity of the synthesis of the optimum scanning antennas, it was repeatedly noted that for two-mirror antennas, if this is not parabola with hyperbola or ellipse, in analytical form it is impossible to record wave front after reflection from two mirrors. All the more it is not possible to analytically express its deviation from standard plane; thus, from the computation of gradient and determination of the direction of rapid descent it is necessary to refuse.

It may be, it goes without saying, that is used the method of the numerical determination of dependences $\frac{\partial\delta L}{\partial y_1}, \frac{\partial\delta L}{\partial m}, \ldots$ with following interpolation results and already from the interpolated curves calculate gradient. However, it seems to us that this path not

justified is laborious and will not ensure the desired shortening in the estimated time. Therefore simpler can prove to be the idea of the method of Kachmazh which he proposed for the solution of the system of simultaneous linear equations and is schematically depicted on Fig. III-1 and III-2 [10].

It is appropriate to note that, speaking about the method of Kachmazh, we we do not replace with it the methcd of the dynamic programming: we are speaking about the organization of the method of sorting within dynamic programming. Understanding the method of dynamic programming very widely, as this is made, for example, in [12], we do not disrupt the essence of this method by this approach to the organization of the elementary operation of synthesis.

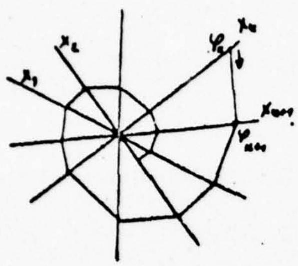
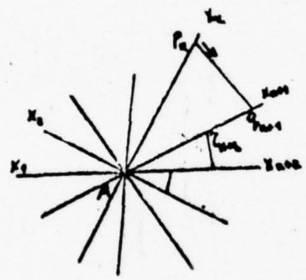


Fig. III-1.

Fig. III-2.



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In the most complex cases, to which belongs and our task, can go not about the complete application/use of a method of Kachmazh, but about the use of fundamental principles of this method. So, instead of the hyperplanes it is necessary to examine the hypersurfaces on which lie/rest on curve, that correspond to derivatives of function ΔL (or $\Delta \ell$)in terms of the parameters. This idea can be sufficiently fruitful not the stage of the optimization (not synthesis) of parameters of the initial antenna, for example aplanatic, which forms then base for the synthesis of optimum in the

assigned sense antenna. In this case, the construction of axisymmetric two-mirror antenna makes it possible to sufficiently easily find some particular directions of rapid descent, i.e., the directions, which coincide with the direction of rate of change in the function in its separate parameters. For example, the optimization of the position of focal point in the known position of standard plane front at the output of antenna can be brought tc successive motion along function x_1 when $y_1 = \mathrm{const}$ (Fig. III-3). The same concerns the optimization of relation f/d with given diameter and maximum phase error (Fig. III-4). Here, both the easy to see that the direction of rapid descent only one of the two possible is defined as direction in which the aberration is less than its preceding/previous value.

However, even in simplest case already during the optimization of the parameters of supporting/reference antenna we encounter s Dependence of its properties from several variatles, i.e., task passes to the category of the multidimensional tasks of control.

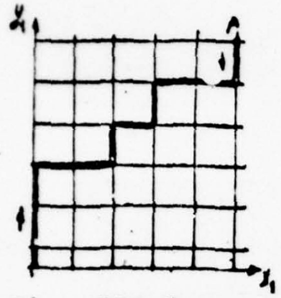

Fig. III.3.

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If the state of system is described by one variable, then back a can be completely correctly solved with the aid of table in the terms of Bellman's dynamic programming [14].

Число оставшихся стадий (1)	Состояние систем (2)			
	0	x_1	x_2	...
I	$f_1(0)$	$f_1(x_1)$	$f_2(x_2)$	...
2	$f_2(0)$	$f_2(x_1)$	$f_2(x_2)$	...
.	.	.	.	
.	.	.	.	
.	.	.	.	
N	$f_N(0)$	$f_N(x_1)$	$f_N(x_2)$	...

Key: (1). Number of remaining stages. (2). State of system.

Here $f_N(x)$ - is the maximum income, obtained for N of the remaining stages of process, which begins from state x and which takes place in accordance with the optimal strategy:

$$f_N(x) = \max\left[g(y_N) + f_{N-1}(x - y_N)\right]; \qquad (\text{III}.1)$$

$$0 \leq y_N \leq x;$$

$$f_1(x) = \max\left[g(y_1)\right] = g(x); \qquad (\text{III}.2)$$

$$0 \leq y_1 \leq x.$$

If it is necessary to utilize two variables, it is possible to construct grid in the space of two phase coordinates, for example x и z.

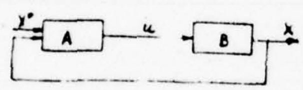

Fig. III.4.

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It is obvious, the tasks of by two variables can require the considerably larger space of the memory of machine and much larger time for solution, than task with one by variable. It is important also to note that the state of system can be described by one variable, but can exist certain number of controlling variables which affect the value of objective function and do not affect the state of system. In our case the picture another: there is a whole series of the governing parameters which affect objective function, without changing the state of system; they are this involved, for example, the coordinate of source in focal curve. At the same time there are and such steering functions which affect the state of system and the objective function, and also some governing parameters. This function, obviously, is the equation of characteristic curve: it affects the state of antenna (form of its surfaces is changed) and to the form of focal curve with scanning.

On the present level of knowledge of task with many variables, they substantially limit the sphere of the applicability of the methods of dynamic programming, especially in the fairly complicated methods when it is necessary to construct multidimensional grids.

Число оставшихся стадий (1)	Состояние системы (2)		
	x_1z_1	x_1z_2	$x_1z_3, \ldots, x_3z_3$
I	$f_1(x_1z_1)$	$f_1(x_1z_2)$	$f_1(x_1z_3), \ldots, f_1(x_3z_3)$
2	$f_2(x_1z_1)$	$f_2(x_1z_2)$	$f_2(x_1z_3), \ldots, f_2(x_3z_3)$
.	.	.	. .
.	.	.	. .
.	.	.	. .
N	$f_N(x_1z_1)$	$f_3(x_1z_2)$	$f_N(x_1z_3), \ldots, f_N(x_3z_3)$

Fig.

Key: (1). Number of remaining stages. (2). State of system.

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§1. Special feature/peculiarities of the elementary operation of the synthesis of monofocal two-mirror antenna with the fixed/recorded sector of scanning.

Synthesis in principle can be conducted by both the by radiation method and the method of needle-shaped variations.

In first method the antenna as such at the beginning of synthesis not at all exists and is created only in the process of synthesis. With an entire temptation of this method, it is characterized by large labor expense, especially if be several control functions, by low efficiency due to the presence of uncontrolled sections and by randomness in the selection of initial conditions at the i stage of the first stage of synthesis.

The second method, which we will use subsequently, makes it possible to considerably shorten computational difficulties, in

particular because in initial stage can be undertaken certain antenna, which thereby or by different of the already known methods is led to that perfection, which it is possible to achieve during its use.

In accordance with last/latter consideration the synthesis of the optimum scanning antenna is conveniently begun with certain aplanatic or bifocal antenna: Aplanatic antenna can be used as fundamental principle with synthesis of the scanning monofocal antennas, but known bifocal - with the synthesis of bifocal astigmats which do not have a precise focus.

Aplanatic two-mirror antennas have characteristic curve in the form of circumference and are described by the equations:

$$\frac{dr}{d\varphi} = r\,\frac{f\sin\varphi + 2\,\mathrm{tg}\,\frac{\varphi}{2}(d-r)}{2\left(d - f\sin^2\frac{\varphi}{2}\right)};$$

$$x = \frac{4d(r-d) + f(\varphi)\sin^2\varphi\,(f-2r)}{2[2d - r(1-\cos\varphi)]};$$

$$y = f\sin\varphi.$$

Let us accept the equations of this antenna for fundamental principle with the synthesis of two-mirror antenna with the oscillation of radiation pattern in sector $\pm\theta°$ from axis.

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In this case, we will be restricted at first to the determination of the optimum parameters of antenna in the case of the location radiation patterns the edge of sector, i.e., at the point, which corresponds to the beam deflection of angle of 6°. By known methods can be found the optimum position of source in aplanatic antenna $x_1 y_1$ for this deviation of diagram and the process of synthesis in each stage will be following: for datum optimum $x_1 y_1$ we begin needle-shaped variations in characteristic curve (circumference), beginning about any part of it, for example from point $y=0$ or $y = y_{max}$.

The semantic part of the calculation, the optimization of characteristic curve, actually consists of the joint solution of the system of nonlinear equations, which forms the aberrations of the separate ray/beams:

$$\left.\begin{aligned}
\Delta l_1 &= \Delta l_1\left[x_{2.3.}, y_{2.3.}, z_{2.3.}, \varphi_{2.3.}, z_{6.3.}, \varphi_{6.3.}, f(\varphi)_{2.3.}, f(\varphi)_{6.3.}\right];\\
\Delta l_2 &= \Delta l_2\left[x_{2.3.}, y_{2.3.}, z_{2.3.}, \varphi_{2.3.}, z_{6.3.}, \varphi_{6.3.}, f(\varphi)_{2.3.}, f(\varphi)_{6.3.}\right];\\
&\vdots\\
\Delta l_n &= \Delta l_n\left[x_{2.3.}, y_{2.3.}, z_{2.3.}, \varphi_{2.3.}, z_{6.3.}, \varphi_{6.3.}, f(\varphi)_{2.3.}, f(\varphi)_{6.3.}\right]
\end{aligned}\right\} \qquad (\text{III}.3)$$

during the limitations:

$$\left.\begin{aligned}
&y \le y_{max};\\
&d_{min} < d < d_{max};\\
&M_{min} < M < M_{max};\\
&f_{0min} < f_0 < f_{0max};\\
&\varphi_{min} < \varphi < \varphi_{max}.
\end{aligned}\right\} \qquad (\text{III}.4)$$

In system III.3, $x_{\text{г.з.}}, y_{\text{г.з.}}, z_{\text{г.з.}}, f(\varphi)_{\text{г.з.}}$ - are the parameters of the mirror point of the ray/beam in question from main mirror; $\dot{z}_{\text{в.з.}}, \dot{\varphi}_{\text{в.з.}}, \dot{f}(\varphi)_{\text{в.з.}}$ - the parameters of the mirror point examined ray/beam from auxiliary mirror.

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The solution of system must be such so that the obtained values $(x_{\text{г.з.}}, y_{\text{г.з.}}, \ldots)$ would provide the minimum of the standard deviation wave of front from reference plane, i.e., this process, strictly, and forms the variational problem of synthesis. However, the solution of system of equations III.3 taking into account the taken limitations is not rationally of synthesis on the strength of both the complexity of equations themselves and the interdependency of steering functions, as that is shown above. Therefore, taking into account all the presented considerations, we let us turn to the singularly acceptable method of solution - to dynamic programming [13, 14].

Initially it seems that the content of synthesis in the terms of

dynamic programming can be represented by the recursion formula

$$\phi_{n+1}(x) = \min\left\{\phi(U_{n+1}) + \left[\phi_n(U_n) + \phi_{N-(n+1)}(U_n)\right]\right\}. \quad (III.5)$$

Here $\phi_{n+1}(x)$ - the minimum value of the aberrations of an entire antenna on $(n+1)$-ache the stage of the synthesis, which was begun from state x (certain antenna) and which takes place in accordance with the optimal strategy: the vector of steering functions U provides a decrease in the aberrations.

The sense of relationship/ratio III.5 can be expressed as follows. Minimization is conducted by the proper selection U_{n+1} at the arbitrary stage of interval $0 \leq y \leq y_{max}$. During stage U_{n+1} the selection of the values of steering functions $n+1$ is carried out taking into account the presence preceding/previous n - stages (counting from i stage) and N - (n + 1) the subsequent stages at which the values of function U already exist (it remained from the preceding/previous process). Of this, consists the principle of optimum character, which lies at the base of the synthesis of the scanning optical-type antennas.

Specifically, during the first stage is selected such value of the radius-vector of characteristic curve which provides

$$\phi_1(x) = \min\left[\phi_1(u_1) + \phi_{N-1}(U - u_1)\right].$$

Here U is entire resource/lifetime of control, i.e., the permissible class of characteristic curves and the range of variations in the separate zones;

u_1- the resource/lifetime of control in this zone, i.e., the limits of a variation in the radius-vector of characteristic curve in this zone;

$f_1(x)$- is the minimum value of distortions, which can be obtained for an antenna, which is found in state x, if one of its zones (here - the first) changes in accordance with the optimal strategy.

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After computation in accordance with this formula, is reject/thrown the old value of the aberrations of this zone and in the memory of machine remains the new value of the aberrations of this zone and all standard deviations of remaining zones, calculated for a supporting/reference antenna.

Thus, process at each stage is conducted on the basis of considerations about the need for the minimization of aberrations both at this stage and taking into account the values, obtained at

the subsequent stages; by this is eliminated the "myopia" of strategy.

It is appropriate to recall that the classical form of the recurrent relationship/ratio of dynamic programming takes the form

$$f_{n+1}(x) = \min[g(y) + f_n(x-y)], \qquad (\text{III}.6)$$

i.e. at each stage of minimization, process consists in the fact that the selection of control pressure y at this stage must minimize functional taking into account those who were obtained at the beginning of this stage of results $f_n(x-y)$. In other words, dynamic programming deals every time with countershaft at this stage plus the initial effect, which established as a result of all previous stages: there is no account of subsequent results and be it cannot, since the control system exists in time and at each torque/moment usually there is information about its behavior only in passed and present torque/moments. In this,, as can easily be seen that consists a difference in the principle of optimum character, which lies at the base of the synthesis of two-mirror antenna from the principle of R. Bellman's optimum character.

The elementary computational operation of synthesis can be about small changes in the identical both at the stage of optimization and at the second consisting stage, i.e., at the stage strictly of

synthesis.

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Let us examine the procedure of computations at the stage of synthesis by the method of needle-shaped variations. Let us assume that the antenna works on reception/procedure and on auxiliary mirror fall the ray/beams of source after reflection from main mirror (Fig. III.5).

The typical procedure of synthesis is a variation of radius vector of characteristic curve at points with ordinate $+y$ and $-y$. In this case,, obviously, will be changed the parameters of mirrors first of all at points C, D, it is more precise, on annular sections by diameter $AC = 2y$ and BD on main and auxiliary mirrors. Therefore function ΔL for entire aperture from new characteristic curve will virtually differ from supporting/reference antenna only by the value of aberrations along the ray/beams of form I, II, III, IV, which fall on section AS and VD.

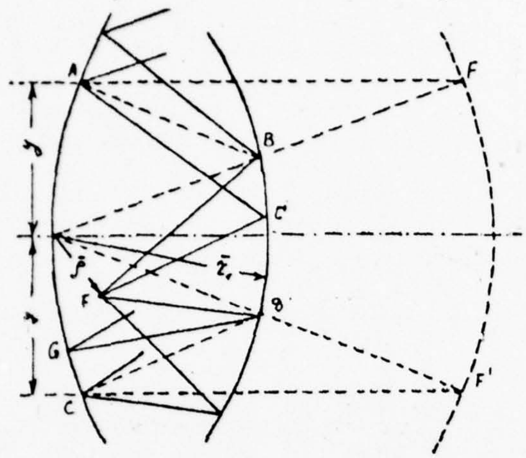

Fig. III.5.

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The ray/beams, reflected from ring BD and which belong to front H, are assigned as follows:

$$\bar{H}_1 = (C_1 - |\bar{\tau}_1 - \bar{\rho}_1|)\,\bar{\xi}_{\tau_1} + \bar{\tau}_1; \qquad (\text{III}.7)$$

$$|\tau_1 - \bar{\rho}_1| = \sqrt{(\tau_1\cos\varphi_1 - x_1)^2 + (\tau_1\sin\varphi_1\cos\alpha_1 - y_1)^2 + (\tau_1\sin\varphi_1\sin\alpha_1)^2};$$

$$\bar{\xi} = \bar{\rho}_1 - 2\bar{\eta}_1(\bar{\eta}_1\bar{\rho}_1);$$

$$\bar{\eta}_1 = \bar{i}\cos(\varphi_1 - \beta_1) - \bar{j}\cos\alpha_1\sin(\varphi_1 - \beta_1) + \bar{k}\sin\alpha_1\sin\alpha_1\sin(\varphi_1 - \beta_1);$$

$$\bar{\rho}_1 = \bar{i}(\tau\cos\varphi_1 - x_1) + \bar{j}(\tau_1\sin\varphi_1\cos\alpha_1 - y_1) + \bar{k}(\tau_1\sin\varphi_1\sin\alpha_1);$$

$$\frac{d\tau_1}{d\varphi_1} = \tau_1\,\frac{f(\varphi_1)\sin\varphi_1 + 2\,\mathrm{tg}\,\frac{\varphi_1}{2}(d - \tau_1)}{2\left(d - f\sin^2\frac{\varphi_1}{2}\right)};$$

ϕ_1 - it is assigned, i.e., $\tau_1\sin\varphi_1 = \mathrm{const}$;

$f(\varphi)$ - is varied;

α_1 - determines point on ring BD in arbitrary section.

The ray/beams, reflected from ring AC and which belong to front H_2, can be found analogously:

$$\left.\begin{array}{l}
\dfrac{x-r_2\cos\varphi_2}{\xi_{2x}}=\dfrac{y-r_2\sin\varphi_2\cos\alpha_2}{\xi_{2y}}=\dfrac{z-r_2\sin\varphi_2\sin\alpha_2}{\xi_{2z}};\\
x=r_1\cos\varphi_1-(y-r_1\sin\varphi_1)\operatorname{ctg}2\gamma_1;\\
y=f(\varphi_1)\cos\alpha_1;\\
z=y\sin\alpha_1;\\
\dfrac{dr_2}{d\varphi_2}=r_2\dfrac{f(\varphi_2)\sin\varphi_2+2\operatorname{tg}\frac{\varphi_2}{2}(d-r_2)}{2\left[d-f(\varphi_2)\sin^2\frac{\varphi_2}{2}\right]};
\end{array}\right\}\quad(\text{III}.8)$$

$$\bar{H}_2=(C_2-|\bar{R}_1-\bar{H}_1|)\,\bar{\xi}_2+R_1;$$

$$\bar{R}_1=\bar{i}x+\bar{j}y+\bar{k}z;$$

$$\bar{\xi}_2=\bar{t}-2\bar{\eta}_2(\bar{\eta}_2\bar{t});$$

$$\bar{\eta}_2=\bar{i}\cos\gamma_1-\bar{j}\cos\alpha_1\sin\gamma_1+\bar{k}\sin\alpha_1\sin\gamma_1;$$

$$\bar{t}^2=i\xi_{2x}+j\xi_{2y}+\bar{k}\xi_{2z};$$

$$\gamma_1=\operatorname{arctg}\frac{y-r_1\sin\varphi_1}{2d-r_1(1-\cos\varphi_1)}.$$

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It is easy to see from Fig. III.5 that the part of the ray/beams of ring VD can exceed the limits of the aperture of main mirror, and for the target/purpose of the fulfillment of inequality $y_{max} \leq \frac{D}{2}$ these ray/beams from examination are eliminated also aberrations for them are not calculated; the same is related also to the part of the ray/beams of ring AC of main mirror, if is superimposed limitation on the size/dimension of auxiliary mirror.

The remaining ray/beams, which do not belong to rings AC and VD, will not virtually change their directions and aberrations for them are determined stereotypically, i.e., they are assigned with certain space in the interval

$$-\varphi_{max} \leq \varphi_1 \leq \varphi_{min}$$

taking into account the fact that $\varphi_1 \neq \varphi_2 (\varphi_1 \leq \varphi_2)$- the wave front, reflected from the auxiliary mirror

$$\bar{H}_1 = (C_1 - |\bar{r}_1 - \bar{\rho}_1|)\bar{\xi}_1 + \bar{r}_1.$$

Wave front, which is reflected from the main mirror

$$H_2 = (C_2 - |\bar{R}_1 - \bar{H}_1|)\bar{\xi}_2 + \bar{R}_1. \qquad \text{(III-9)}$$

Computations by formulas for rings AS and VD, obviously, must be

conducted at each stage for all variations in characteristic curve. Aberrations for the zones which undergo a variation and in all remaining zones can be considered differently. Actually, us in the final analysis interest the sums of the standard deviations of path lengths along the ray/beams in question from standard path. In the formula of aberrations, enter, obviously, the aberrations of the preceding/previous, subsequent zones, and also the zones which are varied on this stage. Aberrations which follow $N-(n+1)$- and preceding/previous n - zones in the process of this needle-shaped variation $(n+1)$ in the - zone are not changed; therefore are possible two ways of the calculations: in the first case the function ΔL is calculated by pillar for entire aperture during each variation $n+1$ in the 1st interval.

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Another path assumes that before beginning of this stage of synthesis are calculated separately all N of standard deviations with respect to the number zones in question and all these values are memorized together with common/general/total function ΔL and part of them is utilized for the further construction of the functions of quality.

After $(n + 1)$ - stage remain $n + 1$ new standard deviations and $N - (n + 1)$ - old, calculated for a supporting/reference antenna,

i.e., occurs the gradual replacement of the values, which are contained in the memory of machine, but a quantity their always is retained. however, this path can be used only in such a case, when we disregard effect n + of the 1st varied stage on one of n - of those who were varied and N-(n + 1) - of the nonvaried stages.

However, strictly speaking, necessary to consider certain change in the parameters of the zones, close to that that is examined at this stage of synthesis. Let us examine again of Fig. III.5. With a variation in the parameters of ring, AC is selected certain direction, for example ray/beam FEA. But if on one of the following stages will be varied the parameters of ring by a radius $r_E \sin\varphi_G$, that direction of beam FEA already somewhat it will change and it does not fall down into point A of main mirror, but this means that the corresponding aberration also to change also in sum n - stages it is necessary to give the appropriate replacements.

Furthermore it is apparent that a variation in the parameters of ring VD will affect the course of ray III and its aberrations; but this ray/beam falls on main mirror at point G, a $y_G < y_{C(A)}$, i.e. this point it belongs to those following by the stages of synthesis.

Thus, strictly speaking, we must each time operate according to the recursion formula

$$f_{n+1}(x)=\min\left\{\left[f_{n+1}(U_{n+1})+\Sigma f_p(U_p)\right]+\left[\Sigma f_n(U_n)+\Sigma f_{N-(n+1)-p}(U)\right]\right\}, \quad (\mathrm{III.10})$$

$\varphi_{n+1}(\upsilon_{n+1})$- is an aberration in a given stage,

$\Sigma\varphi_p(\upsilon_p)$- the sum of the aberrations of those zones on which operates present stage;

$\Sigma\varphi_n(\upsilon_n)$- the sum of those standard deviations on which this stage does not operate (in range from the first to the datum);

$\Sigma\varphi_{N-(n+1)-p}(\upsilon)$- are sum of standard deviations at all subsequent stages, to which thus far still corresponds the supporting/reference antenna, which remained from the preceding/previous stage.

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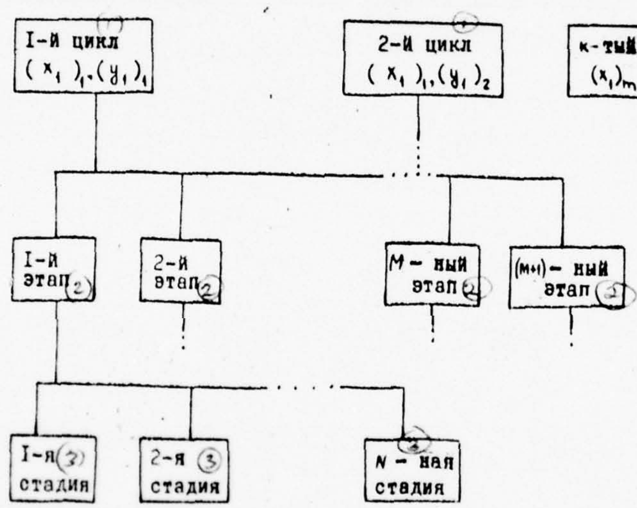
It is hence logical to draw the conclusion that at each stage it can prove to be simpler to calculate the aberrations of entire aperture. The computational apparatus of the synthesis of monofocal two-mirror antenna with the fixed/recorded sector of scanning consists of the step by step application/use of a complex of formulas (III.7) - (III.9) on all N - the stages of each cycle of synthesis. Moreover the functional equation of dynamic programming is actually

faster semantic, I will eat calculated, and is determined the logic of the algorithm of the synthesis which consists of the solution to equations (III.7) - (III.9) during the assigned limitations.

The common/general/total course of synthesis in the simplest case can be represented following block diagram.



Key: (1). cycle. (2). stage. (3). stage.

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Here $(x_1)_1$, $(y_1)_1$ I correspond to the optimum position of irradiator in supporting/reference aplanatic antenna during the deviation of the maximum of radiation pattern of the same angle θ.

Entire process of synthesis, as is evident, it is broken into the separate semantic sections: cycle corresponds to this position of the irradiator for which it is carried cut (M + I) the stages of optimization, but each stage contains in turn, N of the stages whose quantity is connected with the number of intervals to which are divided entire antenna aperture, and also with a quantity of sorted

out coordinates of the points of characteristic curve for each zone.

Strictly, the process of needle-shaped variations contains M of the stages each from which is subdivided into N of stages. After the first stage from N of stages, must be carried cut the second stage also from N of stages for the target/purpose of the explanation of the possible effect of the first sections of characteristic curve on the common picture of aberrations taking into account its new sections which are obtained at the last/latter stages of the first stage. Actually, at the first stages of the first stage a needle-shaped variation in the points, for example in region it occurs in the antenna at whose region γ_{max}, it occurs in the antenna at whose region $\gamma \approx 0$ has circular characteristic curve and an optimum of the i needle-shaped variation (i stage of the i stage) appears itself precisely taking into account this form of characteristic curve.

It is obvious, after I-th stage of synthesis, characteristic curve will no longer be circumference.

Therefore it is expedient to determine, will not change common/general/total result, if we subject again of variation region $\gamma \approx \gamma_{max}$ in the presence of new region $\gamma \approx 0$.

It is obvious, a quantity of stages M must be in the final analysis such so that the aberrations after (M + I) stage would prove to be more than after the m stage.

After are travelled everything (M + 1) of the stages of synthesis, i.e., is finished the i cycle, we change coordinates x_1, y_1. The second cycle in principle can have different duration depending on the degree of the optimum character of coordinates x_1, y_1, obtained as a result of the i cycle.

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If $(x_1)_1, (y_1)_1$ are already optimum and phase error will be minimum, then cycle $[(x_1)_2, (y_1)_1]$ will contain actually only one stage of synthesis. For a complete guarantee it is necessary to conduct also cycle $[(x_1)_3, (y_1)_1]$ with the displacement of focal point in the opposite direction. This cycle with optimum $[(x_1)_1, (y_1)_1]$ will also have only one stage. Thus, if not one of the new values x_1 and y_1 decreases the value of standard deviation, then synthesis can be considered terminal; but if with some x_1 or y_1 phase error it decreases, then for this position of source it is necessary to conduct another a series of stages N-stage process of synthesis in stages by means of needle-shaped variations in characteristic curve.

Thus, after conducting of certain number of cycles for the different positions of source and the identical position of the plane of reference of the ideal wave front, which forms angle θ with vertical line, we will obtain optimum antenna.

The presented pattern of the construction of the elementary operation of synthesis satisfies the fundamental requirements, presented for the algorithms:

a) it is the sufficiently determined sequence of operations, determined by recursion formula, that does not leave the place to arbitrariness, but limitations provide the concreteness of task;

b) elementary operation is mass, since it can serve for the solution of the whole class of the tasks: it is utilized for the synthesis of two-mirror monofocal antennas with any initial antenna of axisymmetric and axially nonsymmetric, and also for the synthesis of lens monofocal antennas;

c) operation is fruitful/successful, i.e., directed, since the fulfillments of equation for $\varphi_{n+1}(x)$ according to the presented diagram, provide the finiteness of task after conducting (K + I) of cycles, the result of this must be optimum antenna.

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§2. Algorithm of the synthesis of monofocal two-mirror antenna with the fixed/recorded sector of scanning.

In this paragraph we will turn directly to the examination of the totality of the calculation formulas, which lie at the base of the synthesis of two-mirror antenna with the optimum parameters. Initially the target/purpose of synthesis is the simplest task - provision for minimum aberrations during the deviation of radiation pattern in monofocal antenna.

Monofocal antenna with the oscillation of radiation pattern, apparently, is only first approximation to the construction of the antenna which provides the scanning/sweep of ray/beam in sector. Actually, satisfaction of Fermat's condition in such antennas means that they form/shape parallel beam only for one position of source; therefore producing pencil beam is fundamental designation/purpose of monofocal antenna, and the possibility of beam swinging only its supplementary property. Of the optimum scanning antenna the phase error in sector must be in the best case of uniform. Before passing to the synthesis of such antennas, let us give the algorithm of the

calculation of the monofocal scanning antennas of particular form.

a) the condition of task. To develop monofocal antenna with the following functional parameters: x_0, y_0- the position of focus on the axis; x_1, y_1- the position of source in focal plane, which corresponds to the deviation of radiation pattern of θ^o from axis. Thus, the target/purpose of synthesis lies in the fact that, obtaining of antenna with focus on the axis, but ensuring smallest possible distortions during the beam deflection of angle $\pm\theta$ from axis, moreover source must be arrange/located at the previously fixed point. Here, it is logical, the discussion concerns variational problem and the minimum of distortions at point x_1, y_1 (focus at point x_0, y_0) is assumed not in absolute, and in extreme sense, i.e., it is not assumed that the aberrations must be equal to zero.

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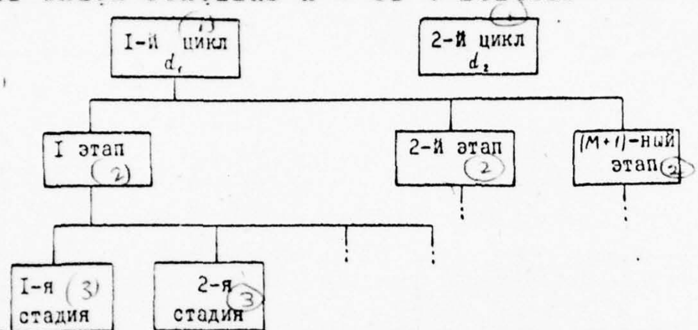
For the synthesis of this antenna, it is possible to use the procedure of the preceding/previous paragraph, the problem can be still simplified, if besides the positions of irradiators is assigned the position of axial focus relative to the apex/vertex main mirror (apical cut M). In this case is realized a variation only in characteristic curve and distance between mirrors.

The block diagram of synthesis consists of N + of 1 cycle, each of which contains M + of 1 stages:

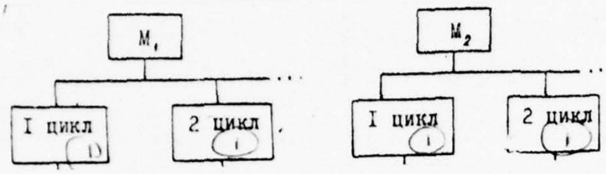


Key: (1). cycle. (2). stage. (3). stage.

Here "stage" corresponds to separate variations in the parameters of this zone, and the group of "stages" is completed by the selection of the optimum parameters of zone and it composes the various "stage" of synthesis.

Each "cycle" of block diagram corresponds to the different axial size/dimension of d: a next series of the stages of synthesis makes it possible to establish/install the mutual effect of variations in characteristic curve and of the axial size/dimension of antenna.

If it is necessary to examine even the effect of the focal cut M, then block diagram must be converted to the form



Key: (1). cycle.

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Here the content of operation "cycle" the same as in preceding/previous block/module/unit to diagram.

Let us examine briefly some of the preliminary optimization which have large value in this problem.

Qualitatively the process of preliminary optimization proceeds as follows. Let there be certain antenna with a diameter of D, the axial size/dimension d_1 and the focal length f_1; the focal curve of this antenna (Fig. III.6), for example, has form A', A', during deviation of radiation pattern to angle θ from axis. In the general case point M with coordinates x_1, y_1 does not lie/rest on this curve, then in the process of synthesis, it is necessary to fit this aplanatic antenna (since the discussion concerns the monofocal

antenna), at whose point x_1, y_1 would lie/rest on focal curved this antenna and its position corresponded to the necessary angle of deflection of radiation pattern, i.e., at first necessary to fit this aplanatic antenna (ℓ_1/d_1, D, M), so that it maximally would approach in its functional parameters [x_0, y_0, x_1, y_1 for $\pm\theta^0$] to by the antenna to system, which is the target/purpose of synthesis. The further process of synthesis is conducted according to the method of needle-shaped variations for the optimization of the form of characteristic curve, an axial size/dimension and a focal cut for the purpose of obtaining minimum distortions at point x_1, y_1.

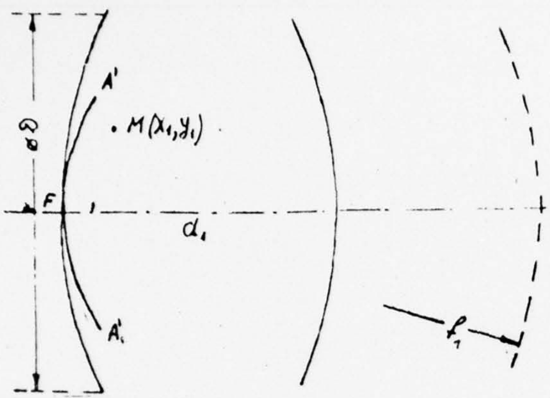

Fig. III.6.

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As a whole the algorithm of synthesis is such.

Condition of the problem of synthesis;

the diameter of antenna D;

the position of irradiator $x_1 y_1$;

the angle of deflection of ray/beam θ.

Limitations:

$d_{min} \leq d \leq d_{max}$; $\varphi_{0min} \leq \varphi_0 \leq \varphi_{0max}$; $M_2 \leq M \leq M_1$.

Initial antenna: two-mirror aplanatic.

The objective function

$$\Delta L = \sum_{1}^{P} \frac{(x_{\Phi} - \bar{H}_{2x})^2 + (y_{\Phi} - \bar{H}_{2y})^2 + (z_{\Phi} - \bar{H}_{2z})^2}{P} . \qquad (\text{III}.11)$$

Here $\bar{H}_2$ is the front, reflected from main mirror,

$$\bar{H}_2 = (C_2 - |\bar{z}_2 - \bar{H}_1|)\bar{\xi}_{z_2} + \bar{z}_2 , \qquad (\text{III}.12)$$

$\bar{H}_1$ - the front, reflected from auxiliary mirror,

$$\bar{H}_1 = (C_1 - |\bar{z}_1 - \bar{\rho}_1|)\bar{\xi}_{z_1} + \bar{z}_1 , \qquad (\text{III}.13)$$

$x_{\Phi}, y_{\Phi}, z_{\Phi}$- Coordinates P of the points of standard plane;

$\bar{z}_1$- the surface of auxiliary mirror;

$\bar{z}_2$- the surface of main mirror;

$\bar{\xi}_{z_1}$- standard to surface of auxiliary mirror;

$\bar{\xi}_{z_2}$- is normal to the surface of main mirror;

$\bar{\rho}_1$ - the frontal surface of the wave, which falls to auxiliary mirror from source with coordinates

The stages of needle-shaped variations consist of consecutive calculation and the estimate/evaluation of the aberrations of all sections, including of those that belong to supporting/reference antenna or the antenna, obtained in the preceding/previous cycle or stage (only calculation), and also the sections for which are realized the variations (calculation and the estimate/evaluation of aberrations).

During use on the first cycle as the supporting/reference aplanatic antenna of aberration during the first stage of the first stage, they consist of the aberrations of this antenna.

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Coordinates of wave front at output from the antenna $x_\varphi, y_\varphi, z_\varphi$:

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$$\left.\begin{aligned}
x_\phi &= \frac{\xi_{2x}\left\{H_{2x}\big|_{y=0}\cos\theta - \left(H_{2y}\big|_{y=0} - H_{2y}\right)\sin\theta\right\} - \xi_{2y}H_{2x}\sin\theta}{\xi_{2x}\cos\theta + \xi_{2y}\sin\theta};\\
y_\phi &= \frac{\xi_{2y}}{\xi_{2x}}\left(x_\phi - H_{2x}\right) + H_{2y};\\
z_\phi &= \frac{\xi_{2z}}{\xi_{2y}}\left(y_\phi - H_{2y}\right) + H_{2z};
\end{aligned}\right\} \quad (\text{III}.14)$$

$$\left.\begin{aligned}
H_{2x} &= \left(c_2 - |\bar{z}_2 - \bar{H}_1|\right)\xi_{2x} + x;\\
H_{2y} &= \left(c_2 - |\bar{z}_2 - \bar{H}_1|\right)\xi_{2y} + y;\\
H_{2z} &= \left(c_2 - |\bar{z}_2 - \bar{H}_1|\right)\xi_{2z} + z;
\end{aligned}\right\} \quad (\text{III}.15)$$

$$\left.\begin{aligned}
\xi_{2x} &= \xi_{1x} - 2\left(\eta_{2x}\xi_{1x} + \eta_{2y}\xi_{1y} + \eta_{2z}\xi_{1z}\right)\eta_{2x};\\
\xi_{2y} &= \xi_{1y} - 2\left(\eta_{2x}\xi_{1x} + \eta_{2y}\xi_{1y} + \eta_{2z}\xi_{1z}\right)\eta_{2y};\\
\xi_{2z} &= \xi_{1z} - 2\left(\eta_{2x}\xi_{1x} + \eta_{2y}\xi_{1y} + \eta_{2z}\xi_{1z}\right)\eta_{2z};\\
\eta_{2x} &= -\cos\gamma_2;\\
\eta_{2y} &= \cos\alpha_2\sin\gamma_2;\\
\eta_{2z} &= \sin\alpha_2\sin\gamma_2;
\end{aligned}\right\} \quad (\text{III}.16)$$

$$|\bar{z}_2 - \bar{H}_1| = \sqrt{(H_{1x}-x)^2 + (H_{1y}-y)^2 + (H_{1z}-z)^2}; \quad (\text{III}.17)$$

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$$\left.\begin{aligned} H_{1x} &= (c_1 - |\bar{r}_1 - \bar{\rho}_1|)\xi_{1x} + x_2; \\ H_{1y} &= (c_1 - |\bar{r}_1 - \bar{\rho}_1|)\xi_{1y} + y_2; \\ H_{1z} &= (c_1 - |\bar{r}_1 - \bar{\rho}_1|)\xi_{1z} + z_2; \end{aligned}\right\} \quad (\text{III}.18)$$

$$\left.\begin{aligned} \xi_{1x} &= p_{1x} - 2(\eta_{1x}p_{1x} + \eta_{1y}p_{1y} + \eta_{1z}p_{1z})\eta_{1x}; \\ \xi_{1y} &= p_{1y} - 2(\eta_{1x}p_{1x} + \eta_{1y}p_{1y} + \eta_{1z}p_{1z})\eta_{1y}; \\ \xi_{1z} &= p_{1z} - 2(\eta_{1x}p_{1x} + \eta_{1y}p_{1y} + \eta_{1z}p_{1z})\eta_{1z}; \end{aligned}\right\} \quad (\text{III}.19)$$

$$\left.\begin{aligned} p_{1x} &= \frac{r_1\cos\varphi_1 - x_1}{|\bar{r}_1 - \bar{\rho}_1|}; \\ p_{1y} &= \frac{r_1\sin\varphi_1\cos\alpha_1 - y_1}{|\bar{r}_1 - \bar{\rho}_1|}; \\ p_{1z} &= \frac{r_1\sin\varphi_1\sin\alpha_1}{|\bar{r}_1 - \bar{\rho}_1|}; \end{aligned}\right\} \quad (\text{III}.20)$$

$$\left.\begin{aligned} \eta_{1x} &= \cos(\varphi_1 - \beta_1); \\ \eta_{1y} &= \cos\alpha_1\sin(\varphi_1 - \beta_1); \\ \eta_{1z} &= \sin\alpha_1\sin(\varphi_1 - \beta_1); \end{aligned}\right\} \quad (\text{III}.21)$$

$$\left.\begin{aligned} |\bar{r}_1 - \bar{\rho}_1| &= \sqrt{(r_1\cos\varphi_1 - x_1)^2 + (r_1\sin\varphi_1\cos\alpha_1 - y_1)^2 + (r_1\sin\varphi_1\sin\alpha_1)^2}; \\ \beta_1 &= \operatorname{arctg}\frac{1}{r_1}\frac{dr_1}{d\varphi_1}; \quad \frac{dr_1}{d\varphi_1} = r_1\frac{f\sin\varphi_1 + 2\operatorname{tg}\frac{\varphi_1}{2}(d - r_1)}{2\left[d - f\sin^2\frac{\varphi_1}{2}\right]}; \end{aligned}\right\} \quad (\text{III}.22)$$

$$r_1 = \frac{d\left(\cos\frac{\varphi_1}{2}\right)^{\frac{2d}{f-d}}}{\sin^2\frac{\varphi_1}{2}\left(\cos\frac{\varphi_1}{2}\right)^{\frac{2d}{f-d}} + \left(1 - \frac{f}{d}\sin^2\frac{\varphi_1}{2}\right)^{\frac{f}{d-f}}}. \quad (\text{III}.23)$$

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Calculation begins against computations r_1 (III.23) at each space φ_1 $(-\varphi_{min} \leq \varphi \leq +\varphi_{max})$, result is valid for all combinations θ, x_1, y_1.

It is convenient to select constant C_1 in the form of sum

$$C_1 = \sqrt{(r_1 \cos\varphi_1 - x_1)^2 + (r_1 \sin\varphi_1 \cos\alpha_1 - y_1)^2 + (r_1 \sin\varphi_1 \sin\alpha_1)^2} + 0{,}05d,$$

which is calculated when $\varphi_1 = \varphi_{1max}$, $\alpha_1 = 180°$ only with change of x_1 and y_1 and does not depend on α_1.

α_1 - the angle, which determines the position of section n auxiliary mirror and together with angle φ_1 assigning is the position of point on this mirror in space. x_1 and y_1 is assigned the position of source, carried out from focus.

The computation of the coordinates of the intersection of ray/beam with main mirror (x_2, y_2, z_2) is conducted as follows: for this combination θ, x_1, y_1, α_1, φ_1 they are calculated for different α_2 in interval $\varphi_{2min} \leq \varphi_2 \leq \varphi_{2max}$:

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$$\left.\begin{aligned} y_2 &= d \sin \varphi_2; \\ x_2 &= r_2 \cos \varphi_2 - (|y_2| - r_2 \sin \varphi_2) \operatorname{ctg} 2\gamma_2; \\ z_2 &= y_2 \sin \alpha_2, \end{aligned}\right\} \quad (\text{III}.24)$$

where

$$\gamma_2 = \operatorname{arc\,tg} \frac{y_2 - r_2 \sin \varphi_2}{2d - r_2(1 - \cos \varphi_2)};$$

r_2 is determined (III.21) with support/socket φ_2 instead of φ_1

Obtained values (III.24) are substituted into the equation

$$\frac{x_2 - x_1}{\xi_{1x}} = \frac{y_2 - y_1}{\xi_{1y}} = \frac{z_2 - z_1}{\xi_{1z}}. \quad (\text{III}.25)$$

where

$$\begin{aligned} x_1 &= r_1 \cos \varphi_1; \\ y_1 &= r_1 \sin \varphi_1 \cos \alpha_1; \\ z_1 &= r_1 \sin \varphi_1 \sin \alpha_1. \end{aligned}$$

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If equation (III.25) is satisfied, then x_2, y_2, z_2 there is really/actually the coordinate of the point of intersection of ray/beam with main mirror.

The recursion formula

$$\phi_{n+1}(x) = \min\left[\phi_{n+1}(U_{n+1}) + \phi_{N-(n+1)}(U - U_{n+1})\right]; \quad \text{(III.26)}$$

$$\phi_0 \leq \phi_{0max}.$$

It is logical to assume that preliminarily from these formulas are calculated the aberrations of entire aperture. However, already after variations during the first stage, the sections in question will not be determined by the circular form of characteristic curve. Actually, during the first stage of the point of characteristic curve, they will take the form:

$$y = y_0;$$
$$x = \phi(\varphi_{y_0}) \cos \varphi_{y_0}.$$

and then

$$y = y_0 \pm \Delta y; \quad \text{(III.27)}$$
$$x = \phi(\varphi_{y_0} \pm \Delta y) \cos \varphi_{y_0} \pm \Delta y.$$

if optimum is not the point of initial characteristic curve.

The coordinates of the varied section, obviously, can be assigned in a most diverse manner, but in the majority of the practical cases is more preferable such method of the assignment of the varied points, which easily can be expressed in analytical form. For example (Fig. III.7), point C belongs to varied section of antenna, and to A and B - point of characteristic curve, belonging

neighboring sections. Point C is characterized by angle φ and by ordinate y_c ; is obvious, the only limitation, which can be superimposed to the coordinates of point C

$$\varphi_A < \varphi < \varphi_B,$$

i.e. the position of point C is limited to the angle of AOV.

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Most simply, apparently, it is possible tc assign the coordinates of points C' and C'', if they are projections na and points A and B respectively.

Then the angle φ of the varied section characteristic curve accepts only three values one of which belongs to initial characteristic curve, and two cthers are connected with projections cf points A and B:

$$\varphi_c = \arcsin \frac{y_c}{f(\varphi_c)};$$

$$\varphi_{c'} = \operatorname{arctg} \frac{y_c}{f(\varphi_{\overline{y_c+\Delta}}) \cos \varphi_{\overline{y_c+\Delta}}};$$

$$\varphi_{c''} = \operatorname{arctg} \frac{y_c}{f(\varphi_{\overline{y_c-\Delta}}) \cos \varphi_{y_c-\Delta}}.$$

Therefore the limitations, presented above can be supplemented by the inequality

$$\operatorname{arctg}\frac{y}{f(\varphi_{y+\Delta})\cos\varphi_{y+\Delta}} \geq \varphi \geq \operatorname{arctg}\frac{y}{f(\varphi_{y-\Delta})\cos\varphi_{y-\Delta}},$$

i.e. at each stage angle φ must not exceed the limits of adjacent stationary angles (angles φ_A and φ_B in Fig. III.7).

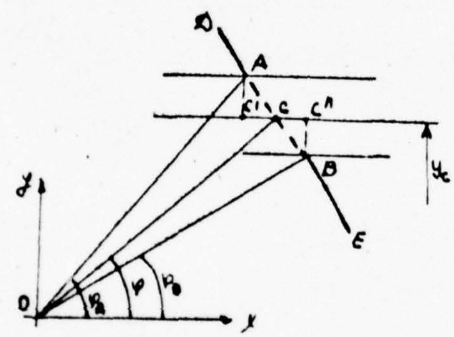

Fig. III. 7.

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A deficiency/lack in this method of the assignment of the coordinates of the varied section is that in the general case cannot be obtained even the local extremum at this stage of needle-shaped variations in view of the limitedness of the range of a change in the variables.

Considerable difficulties can arise already at the first stages of the first stage when as a result of needle-shaped variations instead of, for example, the circular form of characteristic curve it, but at first only its part, is obtained in the form of the dialing/set of the points, not connected by single equation. This, it is logical, it is related also to the surfaces of mirrors. Therefore there can be two possibilities of: either approximating these points of some curve and using during the calculation of it by equation or to memorize entire dialing/set of optimum points. Here it can be noted that during the calculation of the aberrations of the spaced antennas nevertheless is necessary in the final analysis to resort to the discreteness of airfoil/profiles and to the numerical methods of

integration; therefore the memorization of the discrete points of characteristic curve can and not be insurmountable difficulty in the method in question.

The given above formulas for the calculation of aberrations can be, obviously, used also on those stages, at which characteristic curve is not circumference. In the general case characteristic curve can be represented in in the form of a certain curve

$$y = \phi(\varphi)\sin\varphi,$$

and formula (III.8)-(III.23) can be preserved if we represent (III.23) in the form

$$\frac{1}{\tau_1}\frac{d\tau_1}{d\varphi_1} = \tau_1\frac{\phi\sin\varphi_1 + 2\,\mathrm{tg}\,\frac{\varphi_1}{2}\,(d - \tau_1)}{2\left[d - \phi\sin^2\frac{\varphi_1}{2}\right]}.$$

If characteristic curve is assigned in the form of table, then instead of the latter equation it is necessary to utilize recursion formulae.

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Let us note the special feature/peculiarities of the construction of logical operations in those stages in which characteristic curve will be represented by the dialing/set of points.

Before beginning of the first stage of calculation in the memory of machine, is stored the value of the total aberrations of entire aperture of supporting/reference antenna ΔL_0. During the first stage are calculated aberrations ΔL_1, for rings AS and VD(Fig. III.5) with the coordinates of characteristic curve:

$$\begin{aligned} y &= y_A; \\ x &= f(\varphi_{y_A+\Delta}) \cos \varphi_{y_A+\Delta} \end{aligned} \qquad \text{(III.28)}$$

and an entire remaining part of the antenna $\Sigma \Delta l_{oct.1}$. The obtained value $\Delta L_1'$ is compared with supporting/reference ΔL_0 ;if $\Delta L_1' < \Delta L_0$, then of the memory of machine is rubbed ΔL_0, and is memorized $\Delta L_1'$, coordinate (III.28) and an entire remaining part of characteristic curve.

In this case, $\Delta L_1'$ is memorized in the form of the sum of the aberrations of this ring and entire remaining part of the aperture

$$\Delta L_1' = \Delta l_1 + \Sigma \Delta l_{oct.1}.$$

On second stage are calculated the aberrations for rings AS and VD with the coordinates of characteristic curve:

$$\begin{aligned} y &= y_A; \\ x &= f(\varphi_{y_A-\Delta}) \cos \varphi_{y_A-\Delta} \end{aligned} \qquad \text{(III.29)}$$

they are summarized with the aberrations of an entire remaining part

of the antenna $\Sigma \Delta l_{OCT_1}$. If new value $\Delta L_1''$ is lower than previous $\Delta L_1'$, then in the memory of machine remains $\Delta L_1'$, coordinate (III.29) and an entire remaining part of characteristic curve.

At the third stage are calculated the aberrations of the ring

$$y = y_A;$$
$$x = f(\varphi_{y_A}) \cos \varphi_{y_A} \qquad (III.30)$$

and of an entire remaining part, including (III.29). New sum $\Delta L_1'''$ is compared with $\Delta L_1''$ and is selected smaller value, that also is the result of this stage.

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Thus, on each section of aperture are calculated three values of the aberrations of this ring:

$$\Delta l_N^m, m = 1, 2, 3$$

(they are stored in the memory of machine only on the extent/elongation of this stage) and aberrations of an entire remaining part of the antenna

$$\Delta l_{N\,OCT},$$

which are calculated one time for each interval of aperture (each interval it corresponds to three stages of synthesis), $\Delta l_{N\,OCT}$ it is

stored in the memory for the extent/elongation of three stages, which correspond to one interval of aperture.

Total aberration at this stage is equal to:

$$\Delta L_N^m = \Delta l_N^m + \Delta l_{N\,\text{ост}};\quad N = 1, 2, \ldots, M+1;\quad m = 1, 2, 3. \qquad \text{(III.31)}$$

In accordance with that which was presented in the memory of machine, are stored the aberrations in the form of the terms of sum (III.31) and of the coordinates of the optimum points of characteristic curve or the equation of its part, if the discussion concerns the unfinished first stage and there is at least a partially supporting/reference antenna, used as the basis of synthesis.

This formulation of the logical operations of the algorithm of synthesis, obviously, is not the only possibility. For example, as noted above, at each stage of synthesis, can be calculated the aberrations of entire aperture, and then in the memory of machine are stored only initial data and the coordinate of characteristic curve.

Thus, in present paragraph is manufactured the sequence of the calculation formulas whose application/use must ensure the construction of optimum two-mirror antenna with the oscillation of radiation pattern, the deviation of diagram of this angle θ corresponding the location of source at point x, ψ, i.e., is comprised

numerical-logical algorithm, which is of the consecutive execution of arithmetic operations (calculation of aberration) with the subsequent evaluation of the obtained results for each, the stage of synthesis (logical operation) and the selection of the most optimum parameters both the separate sections and the antenna of system as a whole.

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b) the following possible task of synthesis consists of the development of antenna also with the fixed/recorded angular sector of scanning, but already with the predetermined trajectory of the displacement of source. Term "predetermined trajectory of the motion of source" can have the different treatment: it is possible to count that is assigned the equation of trajectory (straight line, circle) and its geometric dimensions, and it is possible to assume that is given only equation of trajectory, and it it is geometric size/dimensions (for example, the position of source at the edge of sector) they are not specified. in this case, can be assigned following parameters: the diameter of antenna, axial size/dimension, size/dimensions the sector of the oscillation of beam and the characteristic points of focal curve. These points can correspond to the position of source on the axis (since it is considered monofocal antenna) and to its position during the deviation of beam of maximum angle. However, assigning the position of source at the edge of the

sector of scanning, it is possible to considerably impoverish task, since the possibilities of a variation in characteristic curve in large measure will come to a variation only in form of characteristic curve without a substantial change in the paraxial focus f_0 of antenna. Therefore in the general case it is expedient to assign only position of source on the axis and the angle of deflection of ray/beam $\pm\theta$, but its position at the edge of sector will be determined into the process of synthesis.

Condition of the task of the synthesis:

the diameter D;

the focal curve $x_1 = f(y_1)$,

the sector of the scanning $0° \leq \theta \leq \theta_{max}$;

focal of cutting. M.

Limitations

$$f \geq f_0,$$
$$M \geq x_1 \geq 0, \quad M \geq 0,$$
$$d_{min} \leq d \leq d_{max};$$

at each stage y angle φ

$$\operatorname{arctg} \frac{y}{f(\varphi_{y+\Delta}) \cos \varphi_{y+\Delta}} \geq \varphi \geq \operatorname{arctg} \frac{y}{f(\varphi_{y-\Delta}) \cos \varphi_{y-\Delta}}.$$

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Calculation formulas - (III.8)-(III.24) taking into account (III.25)-(III-27).

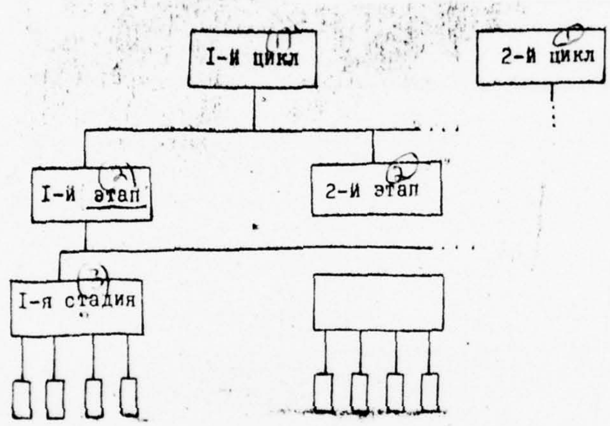
One should especially touch the question concerning the estimate/evaluation of the aberrations when is assigned focal curve, but not isolated point of the location of source. It is easy to see that prevailing will be the requirement not for the minimality of aberrations for the particular angles of deflection of ray/beam, but, for example, the minimum of the average value of the root-mean-square aberrations of entire aperture, calculated for the optimum positions of source in focal curve.

According to approximation method in the space of strategies, as the basis of synthesis can be placed certain known antenna whose fundamental parameters are maximally approximated to the parameters of the which interests us antenna. Let us use, as before, as initial aplanatic antenna with the same size/dimensions d, D, x_0, y_0.

It is assumed also that by the known methods of the optimization of the parameters of aplanatic antennas is found the antenna with

minimum aberrations, i.e., is found the radius-vector φ_0 of characteristic curve (circumference) at assigned d, D, x_0, y_0.

The procedure of synthesis is determined by the following block diagram:



Key: (1). cycle. (2). stage. (3). stage.

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Here each stage - is separate needle-shaped variation in state space. The very complex and laborious result of each variation is the selection of the optimum value of the varied radius-vector of characteristic curve: each variation can be estimated (it is accepted

or reject/thrown) only after the calculation of the aberrations of entire aperture for certain position of source. The realization of stated problem requires the expansion of the space of rough estimate, since it is necessary to consider that it is assigned the whole focal curved, but not one point of the location of irradiator (increase in the dimensionality of task). Therefore to each stage it corresponds as many the computations of the aberrations of entire aperture, as separate positions of sources will be isolated in focal curve (small squares on block diagram). Each stage of block diagram means that for this zone selected optimum value of coordinates taking into account the presence of all remaining sections of mirrors and characteristic function which are obtained at the beginning of this stage.

Each cycle contains the separate dialing/set of calculations for all zones of the antenna: the first cycle contains the first variation in the aplanatic antenna, the second cycle contains a variation in this variation and so forth, until next variation leads to an increase in the aberrations in comparison with the preceding/previous antenna.

c) finally, most general problem of the synthesis of the monofocal scanning antenna - the construction of antenna with the minimum aberrations with of the oscillation of radiation pattern in sector $\pm\theta$ without limitations on form, trajectories and on the

position of sources in the process of scanning.

The only concrete definition, which it is expedient to introduce, is assignment of axial size/dimension and position of focus relative to the apex/vertex main mirror. To variations is subject the form of characteristic curve, including the value of paraxial focal length.

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In the process of synthesis, it is necessary to introduce the concept of the minimum of aberrations in the sector of scanning. to consider the average (or root-mean-square) value of aberrations in all sector is necessary, if we assume that is possible such monofocal antenna, at whose aberrations are not proportional to the deviation of diagram from the axis of antenna. Therefore in the general case it is necessary to utilize value

$$\Delta L_c = \frac{\Delta l_1 + \Delta l_2 + \ldots + \Delta l_p}{p} \qquad (\text{III}.32)$$

or

$$\Delta L_{c.k} = \frac{1}{p} \sum_1^p \Delta l_p^2 ,$$

characterizing distortions in all sector of scanning.

Here ΔL_{c} is an average value of aberrations in sector; Δl_p is an aberration at the particular point of the focal curve: p - the number of points in focal curve in which are designed aberration; ΔL_{CK} is the RMS value of aberrations.

In turn,

$$\Delta l_p = \frac{\Delta l_1 + \Delta l_2 + \ldots + \Delta l_n}{n} \qquad (\text{III}.33)$$

or

$$\Delta l_p = \frac{1}{n} \sum_1^n \Delta l_n^2. \qquad (\text{III}.34)$$

where Δl_n are aberrations on separate ray/beams; n is a number of zones to which is broken aperture of antenna.

Aberrations (III.34) correspond to aberrations (III.8) and the singing of formula (III.8)-(III.24) taking into account (III.25)-(III.27) are valid also in present task. conditions and limitations take the form.

It is given: diameter D, the sector of scanning $0^\circ \leq \theta \leq \theta^\circ_{max}$.

Limitations:

$$d_{min} \leq d \leq d_{max};$$
$$f \geq f_0;$$
$$M \geq x_1 \geq 0, \; M \geq 0.$$

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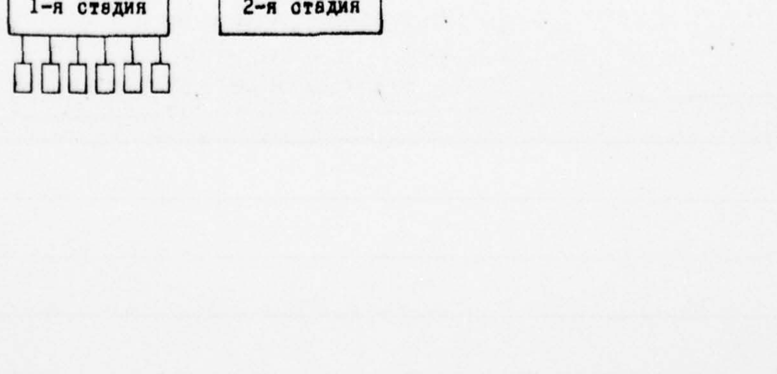
Interval of variations in the angle ϕ at each stage

$$\operatorname{arctg}\frac{y}{f(\varphi_{y+\Delta})\cos\varphi_{y+\Delta}} \geq \varphi \geq \operatorname{arctg}\frac{y}{f(\varphi_{y-\Delta})\cos\varphi_{y-\Delta}}.$$

It is qualitative the process of synthesis, as in the preceding/previous cases, it is convenient to characterize by the block diagram:



Key: (1). cycle. (2). stage. (3). stage.

The first semantic operation is the calculation, which corresponds to the square of "stage." At each stage is varied one coordinate of the zone of antenna in question. For these variations are calculated the aberrations of entire aperture, the source consecutively occupying a series of positions in accordance with the deviation of radiation pattern from axis at angles from 0° up to θ_{max}. In this case, for each position of ray/beam, it is necessary to find the best position of source (point of the best focusing) and the connected groups of small squares correspond to the selection of this point. Thus, each small square corresponds to the calculation of the aberrations of entire aperture for this position of source in this angle of deflection of ray/beam and this variation to coordinate the zone in question.

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The number of groups of small squares corresponds to the number of intervals to which is divided the sector scanning.

Each stage concludes with the selection of a optimum variation in the coordinates of separate zone taking into account aberrations in all sector of scanning.

Calculated cycle corresponds to the development of the new antenna which on its aberrations is better than that that was selected as the base before beginning of the process of synthesis, either it is better than the antenna, obtained as a result of the preceding/previous cycle, i.e., each cycle is a result of a variation in the antenna with new boundary conditions, precisely, from new characteristic curve or new axial focal size/dimension, if variation begins from or zone $(y = 0)$.

§3. Algorithms of the synthesis of non-focal antennas.

As the scanning electromechanical antennas it is most expedient to apply such devices in which because of certain compromise can be obtained the properties, which ensure the greatest informativeness in the process of the execution of the fundamental functions of antenna. If antenna is utilized only for beam swinging, then it is always desirable so that the parameters of radiation pattern little would be changed in all sector of survey/coverage.

a) concerning the problem of the synthesis of antenna with uniform aberrations, it should be noted that the question is not the trivial solution which it can be, for example, is obtained because of the use of a non-optimal form of focal curve.

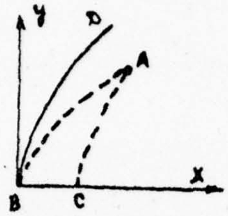

Fig. III.8a.

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Figures III.8a shows (schematically) optimum focal curve AV and other focal curves of VD and AS the moncfocal antenna, which has focus on the axis of symmetry. In the known monofocal antennas of axial symmetry, the dependence of amplification factor on the angle of the oscillation of radiation pattern takes the form of curve I (Fig.III.8a). Apparently, selecting the focal curve (A5 or VD) in Fig. III.8a), it is possible to obtain also in monofocal antenna the dependence $\kappa\cdot\gamma$ on the angle of scanning in the form of straight line II (Fig. III.8b) or of close to it curve. However, it is completely cbvious that this "equalization" according to the worse result is not optimum and it is most desirable to obtain the averaged dependence in the form of straight line III. This direct/straight (or by close to

it curve) must correspond optimum focal curve, i.e., to each angle of deflection must correspond such position of source, which provides the maximum of amplification factor. Last/latter prerequisite/premise means that the requirement for the uniformity of the phase error in sector it cannot satisfy either monofocal, or bifocal antenna, i.e., or one antenna, which forms the collimated light beam in certain position of source.

Important is the question concerning how to select the criterion of optimum character for the modification of antenna in question. On one hand, we must analytically formulate the need of providing the smallness of the error, while on the other hand - its maximum constancy in sector.

It is possible to count that it is necessary to provide the minimum of the maximum deviation of mean error with respect to v s sector ΔL_{x} from the assigned magnitude ΔL.

б)

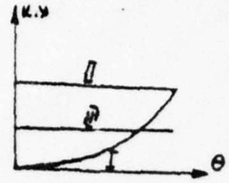

Fig. III-8b.

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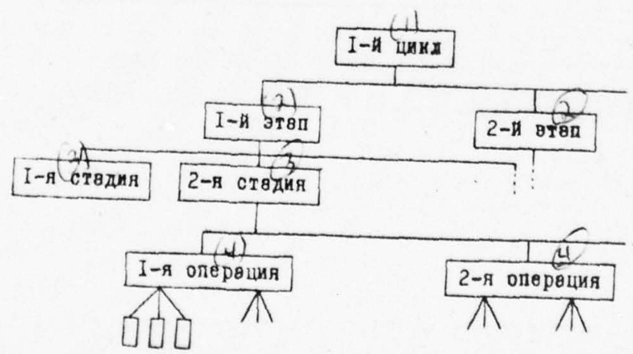
The block diagram of the process of synthesis takes the form:



Key: (1). cycle. (2). stage. (3). stage. (4). Operation.

Here also is conveniently the analysis of block diagram begun

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DESIGN OF OPTIMUM TWO-MIRROR ANTENNAS WITH THE OSCILLATION OF R--ETC(U)

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against the examinations of the sequence of processes under common/general/total name "stage". Each stage corresponds to a separate variation in the parameter ℓ of the zone in question. Since each such variation is conducted by the presence of remaining stationary points (there exists each time entire/all antenna, supporting/reference or obtained on of preceding/previous stages 3, the effectiveness of separate variation is rate/estimated by the computation of aberrations of entire aperture taking into account this variation. Moreover, aberrations must be calculated for entire sector of scanning, broken into the discrete angles of deflection of ray/beam, to each of which it is necessary to find the appropriate optimum position of source (to group of the small squares).

To each value of the radius-vector of characteristic curve ℓ must be placed into conformity the series of the values of angular function $\alpha = \alpha(\phi)$. To each concrete/specific/actual value $\alpha = \alpha(y)$ in this radius-vector of conditional characteristic curve corresponds the process of computations with common/general/total name "operation".

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Here is related the selection of the best positions of source in sector for the discrete angles of the relation of ray/beam (group of

small squares) by the calculation of the aberrations of entire aperture. After will be made the computations, which correspond to all groups of small squares, the conclusion about the effectiveness of a variation in parameters of this zone $f(\Psi)$ and $\alpha(\phi)$ is made on the basis of recurrence formula

$$f_{n+1}(x)=\min\left\{f(U_{n+1})+\left[f_n(U_n)+f_{N-(n+1)}(U_{N-(n+1)})\right]\right\}. \quad (\text{III}.35)$$

Each stage concludes with selection and the memorization of the optimum parameters $f(\varphi)$ and $\alpha(\phi)$ concrete/specific/actual zone, and also the average value of aberrations to sector and degree of deviation of the uniformity of these aberrations from the assigned magnitude. Thus, the number of "stages" is equal to the number of variations in the radius-vector of the separate zones to which is broken antenna aperture (they are implied the circular zones).

The number of "operations", just as stages, not defined, since it depends on the speed of the appearance of an outer limit of the radius-vector characteristic curve and angular function in each zone.

Each "stage" contains entire complex of computations for one zone, and its result are the optimum parameters of this zone.

Each "cycle", as can easily be seen that it is completed in the general case by the synthesis of the new antenna, according to its

scanning properties which exceeds initial antenna.

Let us now move on to the examination of the analytical operations of synthesis.

According to approximation method in the space of strategies, before beginning strictly of synthesis (1st cycle) it is necessary to select the supporting/reference antenna which is maximally close in its parameters and the properties to the which interests us antenna. This antenna can be the antenna, which does not form on output pencil beam. The equations of this antenna are obtained in §2 the first chapter. In the process of synthesis, must be obtained the dependences $\alpha(\phi)$ and $f(\varphi)$ i.e., wave front and the amplitude distribution of compromise antenna.

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As in the case of monofocal antennas, the fundamental function of quality it is

$$\Delta L_{0.n\delta} = \frac{1}{n}\sum_{\phi}^{n}(x_{\phi}-\hat{R}_{2x})^{2}+(y_{\phi}-\hat{R}_{2y})^{2}+(\hat{R}_{2z}-z_{\phi})^{2}. \quad (\text{II}.86)$$

Here $x_{\phi}, y_{\phi}, z_{\phi}$ are coordinates ϕ of the pcints of the standard plane:

$\vec{R}_2$ - the front, reflected from main mirror,

$$\vec{R}_2 = (c_2 - |\vec{r}_2 - \vec{R}_1|)\vec{\xi}_{r_2} + \vec{r}_2 , \qquad (II.37)$$

where in turn, $\vec{R}_1$ - the front, reflected from the auxiliary mirror

$$\vec{R}_1 = (C_1 - |\vec{r}_1 - \vec{\rho}_1|)\vec{\xi}_{r_1} + \vec{r}_1 , \qquad (II.38)$$

$\vec{r}_1$ is a radius-vector of auxiliary mirror; $\vec{r}_2$ is a radius-vector of main mirror; $\vec{\xi}_{r_1}$ is single normal to the surface of auxiliary mirror; $\vec{\xi}_{r_2}$ is single normal to the surface of main mirror; $\vec{\rho}_1$ is a radius-vector of the front, which falls from source about by coordinates x_1, y_1.

The surface of auxiliary mirror is assigned by the differential equation

$$\frac{1}{r_1}\frac{dr_1}{d\varphi_1} = \frac{f(\sin\varphi_1 + \sin\alpha_1) + 3d\sin\alpha_1 - M\,tg\,\alpha_1 - r_1(\sin\varphi_1 - \sin\alpha_1) + tg\frac{\varphi_1 - \alpha_1}{2}\left[(r_1\cos\varphi_1 - \cos\alpha_1) - M + 3d\cos\alpha_1\right]}{r_1(\cos\varphi_1 - \cos\alpha_1) - M + 3d\cos\alpha_1 - tg\frac{\varphi_1 - \alpha_1}{2}\left[f(\sin\varphi_1 + \sin\alpha_1) - r_1(\sin\varphi_1 - \sin\alpha_1)\right] - tg\frac{\varphi_1 - \alpha_1}{2}(3d\sin\alpha_1 - M\,tg\,\alpha_1)} . \qquad (II.39)$$

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For statement in (III.38) it is necessary consecutively to calculate:

$$|\vec{r}_1-\vec{\rho}_1|=\sqrt{(r_1\cos\varphi_1-x_1)^2+(r_1\sin\varphi_1\cos\theta_1-y_1)^2+(r_1\sin\varphi_1\sin\theta_1)^2}\,; \quad (\text{III}.40)$$

$$\left.\begin{aligned}\eta_{1x}&=\cos(\varphi_1-\beta_1);\\ \eta_{1y}&=\cos\theta_1\sin(\varphi_1-\beta_1);\\ \eta_{1z}&=\sin\theta_1\sin(\varphi_1-\beta_1);\end{aligned}\right\} \quad (\text{III}.41)$$

$$\beta_1=\operatorname{arctg}\frac{1}{r_1}\frac{dr_1}{d\varphi_1}\,; \quad (\text{III}.42)$$

$$\left.\begin{aligned}P_{1x}&=\frac{r_1\cos\varphi-x_1}{|\vec{r}_1-\vec{\rho}_1|};\\ P_{1y}&=\frac{r_1\sin\varphi_1\cos\theta_1-y_1}{|\vec{r}_1-\vec{\rho}_1|};\\ P_{1z}&=\frac{r_1\sin\varphi_1\sin\theta_1}{|\vec{r}_1-\vec{\rho}_1|};\end{aligned}\right\} \quad (\text{III}.43)$$

$$\left.\begin{aligned}\xi_{1x}&=P_{1x}-2(\eta_{1x}P_{1x}+\eta_{1y}P_{1y}+\eta_{1z}P_{1z})\eta_{1x};\\ \xi_{1y}&=P_{1y}-2(\eta_{1x}P_{1x}+\eta_{1y}P_{1y}+\eta_{1z}P_{1z})\eta_{1y};\\ \xi_{1z}&=P_{1z}-2(\eta_{1x}P_{1x}+\eta_{1y}P_{1y}+\eta_{1z}P_{1z})\eta_{1z}.\end{aligned}\right\} \quad (\text{III}.44)$$

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Constant c_1 in III.38 we assign in the form

$$c_1 = \sqrt{(r_1\cos\psi_1 - x_1)^2 + (r_1\sin\psi_1\cos\delta_1 - y_1)^2 + (r_1\sin\psi_1\sin\delta_1)^2} + 0.05d. \quad (\text{III}.45)$$

Assigning here $\psi_1 = \psi_{1max}$, $\delta_1 = 180°$, we provide the position of wave front A_1 always in the interval/gap between mirrors. Computations according to III.45 are conducted only upon exchange x_1, y_1 and do not depend on δ_1 and ϕ_1. Angle δ_1 determines the position of section on auxiliary mirror and together with ϕ_1 is assigned the position of the point on this mirror, into which falls the ray/beam of source. Angle δ_1 changes in interval of 0-180°, while angle ϕ_1 - in interval $0 \leq \psi_1 \leq \psi_{1max}$.

The computation of the coordinates of the intersection of ray/beam with main mirror (x_2, y_2, z_2) is conducted as follows: for the assigned combination θ, x_1, y_1, δ_1, ψ_1 they are calculated for equal to δ_2 in interval $\psi_{2min} \leq \psi_2 \leq \psi_{2max}$:

$$x_2 = r_2\sin\psi_2 - \frac{\cos(2\gamma_2 - \alpha)\{f(\sin\psi_2 + \sin\alpha) + 3d\sin\alpha - Mtg\,\alpha - r_2(\sin\psi_2 + \sin\alpha)\}}{\sin(2\gamma_2 - \alpha) + \sin\alpha};$$

$$y_2 = f\sin\psi_2 \; (f\cos\psi_2 - x_2)\,tg\,\alpha; \quad (\text{III}.46)$$

$$z_2 = y_2\sin\delta_2.$$

Here r_2 is determined from (III.39) with substitution ϕ_1 instead of ψ_1 .

Those who were obtained x_2, y_2, z_2 we substitute in

$$\frac{x_2 - x_1}{\xi_{11x}} = \frac{y_2 - y_1}{\xi_{10y}} = \frac{z_2 - z_1}{\xi_{11z}}, \qquad \text{(III.47)}$$

where

$$x_1 = r_1 \cos \psi_1;$$
$$y_1 = r_1 \sin \psi_1 \cos \delta_1;$$
$$z_1 = r_1 \sin \psi_1 \sin \delta_1.$$

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If (III.47) is satisfied, then those who were found x_2, y_2, z_2 are really/actually the coordinates of the point of intersection of ray/beam (III.47) with main mirror.

In terms of the obtained values we determine the front, reflected from the main mirror H_2:

$$\left.\begin{aligned} H_{2x} &= (C_2 - |\bar{r}_2 - \bar{H}_1|)\,\xi_{2x} + x_2, \\ H_{2y} &= (C_2 - |\bar{r}_2 - \bar{H}_1|)\,\xi_{2y} + y_2, \\ H_{2z} &= (C_2 - |\bar{r}_2 - \bar{H}_1|)\,\xi_{2z} + z_2; \end{aligned}\right\} \qquad (\text{III}.48)$$

$$\left.\begin{aligned} \xi_{2x} &= \xi_{1x} - 2(\eta_{2x}\xi_{1x} + \eta_{2y}\xi_{1y} + \eta_{2z}\xi_{1z})\eta_{2x}, \\ \xi_{2y} &= \xi_{1y} - 2(\eta_{2x}\xi_{1x} + \eta_{2y}\xi_{1y} + \eta_{2z}\xi_{1z})\eta_{2y}; \\ \xi_{2z} &= \xi_{1z} - 2(\eta_{2x}\xi_{1x} + \eta_{2y}\xi_{1y} + \eta_{2z}\xi_{1z})\eta_{2z}, \end{aligned}\right\} \qquad (\text{III}.49)$$

$$\left.\begin{aligned} \eta_{2x} &= -\cos(2\gamma_2 - \alpha), \\ \eta_{2y} &= \cos\delta_2 \sin(2\gamma_2 - \alpha), \\ \eta_{2z} &= \sin\delta_2 \sin(2\gamma_2 - \alpha); \end{aligned}\right\} \qquad (\text{III}.50)$$

$$|\bar{r}_2 - \bar{H}_1| = \sqrt{(H_{1x} - x_2)^2 + (H_{1y} - y_2)^2 + (H_{1z} - z_2)^2} \qquad (\text{III}.51)$$

$$\left.\begin{aligned} H_{1x} &= (C_1 - |\bar{r}_1 - \bar{\rho}_1|)\,\xi_{1x} + x_2, \\ H_{1y} &= (C_1 - |\bar{r}_1 - \bar{\rho}_1|)\,\xi_{1y} + y_2, \\ H_{1z} &= (C_1 - |\bar{r}_1 - \bar{\rho}_1|)\,\xi_{1z} + z_2; \end{aligned}\right\} \qquad (\text{III}.52)$$

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The coordinate of plane of reference, which forms angle with y

axis, they take the form:

$$
\left.\begin{aligned}
x_{\phi} &= \frac{\xi_{2x}\left\{H_{2x}\big|_{\varphi_1=0}\cos\theta-\left(H_{2y}\big|_{\varphi_1=0}-H_{2y}\right)\sin\theta\right\}-\xi_{2y}H_{2z}\sin\theta}{\xi_{2x}\cos\theta+\xi_{2y}\sin\theta};\\
y_{\phi} &= \frac{\xi_{2y}}{\xi_{2x}}\left(x_{\phi}-H_{2x}\right)+H_{2y};\\
z_{\phi} &= \frac{\xi_{2z}}{\xi_{2y}}\left(y_{\phi}-H_{2y}\right)+H_{2z}.
\end{aligned}\right\}\quad (\text{III}.53)
$$

Coordinates (III.53) are calculated from entire totality of formulas (III.36)-(III.52) with substitution $\phi_1 = 0$, i.e., standard plane intersects (or it concerns) front Ω_2 at the point of its intersection with the ray/beam of the source which is reflected from auxiliary mirror in point $\phi_1 = 0$.

b) the circuit of non-focal antenna can be placed as the basis of bifocal antenna with the maximum compensation for astigmatism. There is in form an antenna, in which for one position of radiation pattern ($\theta \neq 0$) are obtained minimum aberrations both in the plane of deflection of ray/beam and in perpendicular plane. known bifocal two-mirror and lens antennas [16] were obtained in the form of the bodies of revolution of two-dimensional envelope around the

longitudinal axis. in this case, in one plane of aberration, they are absent, and in perpendicular plane occurs astigmatism. The elimination of aberrations in two planes occurs only in metal-plate lenses with constraint [17]. Two-mirror antenna with such properties cannot be obtained in view of the limitedness of the number of free parameters.

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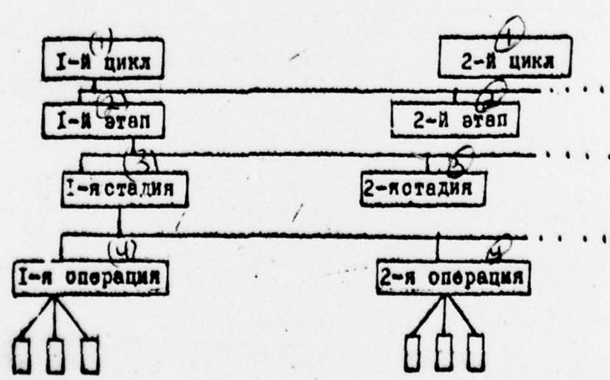
However, the compromise version of antenna with the reduced astigmatism because of certain assumption of coma and spherical aberration can find use as the scanning antenna.

The block diagram of the process of the synthesis of bifocal anastigmatic antenna can be represented in the following form:



Key: (1). cycle. (2). stage. (3). stage. (4). operation.

Here "cycle" - the process of the conversion of supporting/reference antenna (first variation in this antenna); "stage" - a variation in one zone, where enters a variation in the parameters $\phi(\psi)$ and $\alpha(y)$. "stage" corresponds to the separate variation $\phi(\psi)$ to which corresponds a series of variations in the parameters $\alpha(y)$ ("operation"). For this value $\phi(\psi)$ and $\alpha(y)$ in this zone are calculated the aberrations of entire aperture during the deviation of standard plane of angle θ from the axis of antenna.

If is assigned only angle θ and is not assigned the position of source, then it must be found; to this process corresponds one to group of small squares unlike the series of groups, which occurred in the preceding/previous task. Formula and the procedure of their application/use the same as in the case of the synthesis of antenna from uniform phase error in sector, but taking into account the presented special feature/peculiarities.

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